Mathematics 2
D322 12/NAB002

Higher Mathematics

NATIONAL ASSESSMENT BANK
SUPPORT MATERIALS

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Contents

Section 1

Performance Criteria

Performance Criteria

Outcome 1

Use the factor/remainder theorem and apply quadratic theory.

Performance Criteria

(a) Apply the factor/remainder theorem to a polynomial function.
(b) Determine the nature of the roots of a quadratic equation using the discriminant.

Outcome 2

Use basic integration.

Performance Criteria

(a) Integrate functions reducible to sums of powers of x (definite and indefinite).
(b) Find the area between a curve and x-axis using integration.
(c) Find the area between two curves using integration.

Outcome 3

Solve trigonometric equations and apply trigonometric formulae.

Performance Criteria

(a) Solve a trigonometric equation in a given interval.
(b) Apply trigonometric formulae (addition formulae) in the solution of a geometric problem.
(c) Solve a trigonometric equation involving an addition formula in a given interval.

Outcome 4

Use the equation of the circle.

Performance Criteria

(a) Given the centre (a,b) and radius r, find the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$.

(b) Find the radius and centre of a circle given the equation in the form $x^2+y^2+2gx+2fy+c=0$.

(c) Determine whether a given line is a tangent to a given circle.

(d) Determine the equation of the tangent to a given circle given the point of contact.

Section 2

Instrument of Assessment

Instrument of Assessment

Formulae list

Circle The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre *(-g,-f)* and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre *(a,b)* and radius r.

Trigonometric formulae

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

Unit assessment – Mathematics 2 (Higher)

Outcome 1 **Marks**

1 (i) Show that $(x-2)$ is a factor of $g(x) = x^3 + 2x^2 - 13x + 10$

(ii) Hence factorise $g(x)$ fully. **5**

2 Determine the nature of the roots of the equation $2x^2 - 4x + 2 = 0$ using the discriminant. **3**

Outcome 2

3 Find $\int \frac{7}{x^6}\,dx,\ x \neq 0$ **3**

4 The curve with equation $y = x(x^2 - 3x + 5)$ is shown in Diagram 1.

Calculate the shaded area shown in Diagram 1.

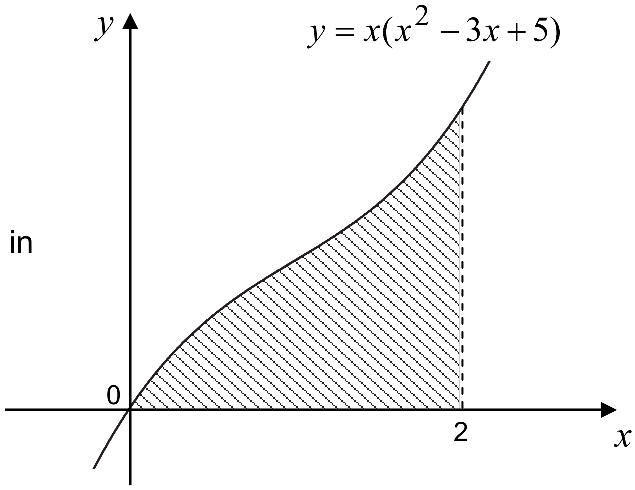

Diagram 1

5

Marks

5 The line with equation $y = x + 3$ and the curve with equation $y = 3 + 5x - x^2$ are shown in Diagram 2.

The line and curve meet at the points where $x = 0$ and $x = 4$.

Calculate the shaded area shown in Diagram 2.

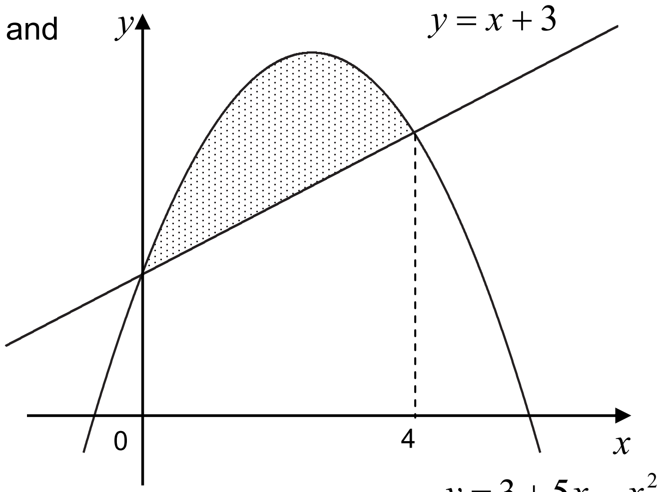

Diagram 2

6

Outcome 3

6 Solve the equation $\sin 2x = \frac{1}{2}$ for $0 < x < \pi$. **2**

7 Diagram 3 shows two right-angled triangles.

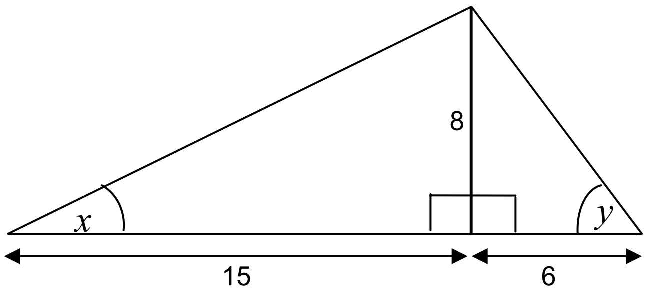

Diagram 3

(a) Write down the values of $\cos x$ and $\cos y$. **2**

(b) Show that the exact value of $\cos(x - y)$ is $\frac{77}{85}$. **2**

Marks

8 (a) Express $\sin x^o \cos 31^o - \cos x^o \sin 31^o$ in the form $\sin(x-a)^o$. **1**

(b) Using the result from (a),

solve $\sin x^o \cos 31^o - \cos x^o \sin 31^o = \frac{8}{11}$ for $0 < x < 360$. **4**

Outcome 4

9 (a) A circle has radius 6 units and centre $(-7,-4)$. Write down the equation of the circle. **2**

(b) A circle has equation $x^2 + y^2 - 4x + 2y - 7 = 0$. Write down the coordinates of its centre and the length of its radius. **2**

10 Show that the line with equation $y = 3x - 2$ is a tangent to the circle with equation $x^2 + y^2 - 8x + 6 = 0$. **5**

11 The point P(2,6) lies on the circle with centre C(5, –3), as shown in Diagram 4.

Find the equation of the tangent at P. **3**

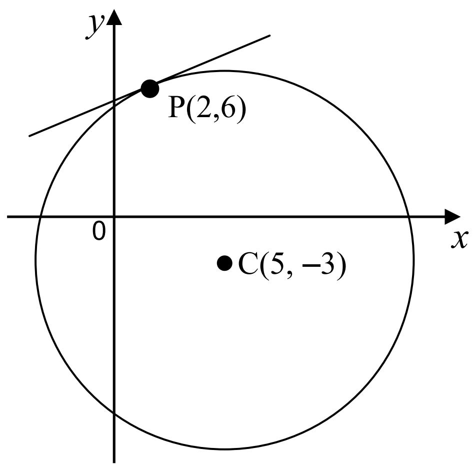

Diagram 4

End of assessment

Section 3

Marking information

Marking information

Test specification grid

The grid below shows how the Outcomes and Performance Criteria are assessed in this Unit assessment.

Topic	PC	Question	Marks	Total	Threshold
Factor/remainder theorem and quadratic theory	1(a) 1(b)	1 2	5 3	**8**	**5**
Basic integration	2(a) 2(b) 2(c)	3 4 4 5	3 5 6	**14**	**10**
Trigonometric equations and formulae	3(a) 3(b) 3(c)	6 7 8	2 4 5	**11**	**7**
Equation of the circle	4(a) 4(b) 4(c) 4(d)	9 9 10 11	2 2 5 3	**12**	**8**

Marking information – Mathematics 2 (Higher)

General marking instructions

1 Marks should be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2 Award one mark for each • (bullet point). Each error should be underlined at the point in the working where it first occurs, and not any subsequent stage of the working.

3 The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided the level of difficulty is approximately similar. Where, subsequent to an error, the working is eased, a deduction of marks(s) should be made.

4 As indicated on the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking information.

5 Do not penalise:

- working subsequent to a correct answer
- omission of units
- legitimate variations in numerical answers
- correct working in the wrong part of a question
- bad form.

6 No piece of work should be scored through without careful checking – even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking information – answers which are widely off beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for marks.

7 No marks should be deducted for careless or badly arranged work.

8 Transcription errors – In general, as a consequence of a transcription error, candidates lose the opportunity of gaining the first accuracy or processing mark.

9 Casual errors – In general, as a consequence of a casual error, candidates lose the opportunity of gaining the first accuracy or processing mark.

10 Acceptable alternative methods of solution can only be given the marks specified in the marking information if the question does not stipulate the method candidates are to use to find the solution. In such circumstances, no marks may be awarded even though the candidate may have obtained the correct answer.

11 In general do not penalise the same error twice in the one question.

Marking signs and abbreviations

It is recommended that markers use the following signs and abbreviations for marking purposes:

✓ Tick when a piece of working is correct and gains a mark.

A cross-tick should be used to indicate ‘correct working’ where a mark is awarded as a result of follow through from an error.

A double cross-tick should be used to indicate correct working which is inadequate to score any marks eg incorrect method which is mathematically correct or eased working.

Underline and cross each error especially those where a mark has been lost.

A tilde should be used to indicate a minor transgression which is not being penalised, eg bad form.

Use a roof to show that something is missing such as a crucial step in the working or part of a solution.

B An upper case B should be used to indicate that you have given the candidate the benefit of the doubt and awarded a mark.

E An upper case E should be used to indicate that the candidate has eased the working as a consequence of an error and that marks have been deducted as a result.

Outcome 1

Qs	Give 1 mark for each •	Illustrations for awarding each mark
1	$g(x)=(x-2)(x-1)(x+5)$	
		Method 1
	$\bullet^1$ Know to use $x=2$	$\bullet^1$ know to use $x=2$
	$\bullet^2$ Complete evaluation and conclusion	$\bullet^2$ $8+8-26+10=0 \Rightarrow x-2$ is a factor
	$\bullet^3$ Start to find quadratic factor	$\bullet^3$ $(x-2)(x^2 \ldots)$
	$\bullet^4$ Complete quadratic factor	$\bullet^4$ $(x+1)(x^2+4x-5)$
	$\bullet^5$ Factorise completely	$\bullet^5$ $(x-2)(x-1)(x+5)$ **stated explicitly**
		OR
		Method 2
		$\bullet^1$ $\begin{array}{r\|rrrr} 2 & 1 & 2 & -13 & 10 \\ \hline & & & & \end{array}$
		$\bullet^2$ $\begin{array}{r\|rrrr} 2 & 1 & 2 & -13 & 10 \\ & & 2 & 8 & -10 \\ \hline & 1 & 4 & -5 & 0 \end{array}$
		$\bullet^3$ $g(2)=0$ so $(x-2)$ is a factor
		$\bullet^4$ (x^2+4x-5)
		$\bullet^5$ $(x-2)(x-1)(x+5)$ **stated explicitly**

Notes

1. In method 1, $\bullet^2$ and method 2, $\bullet^3$, candidates must show some acknowledgement of the resulting '0'. **Do not** accept anything simply as underlining or boxing in the zero.
2. $\bullet^5$ is for the product of the correct three linear factors in any order.
3. In (ii) the correct answer only without working loses $\bullet^3$ and $\bullet^4$.

Outcome 1 (continued)

Qs	Give 1 mark for each •	Illustrations for awarding each mark
2	Equal roots or repeated roots	
	•[1] Know discriminant	•[1] $b^2 - 4ac$ **stated or implied by** •[2]
	•[2] Substitute for *a*, *b* and *c* and evaluate	•[2] $(-4)^2 - 4 \times 2 \times 2 = 0$
	•[3] State nature of roots	•[3] Equal (real) roots **or** repeated (real) roots

Notes:
1. Any other expression masquerading as the discriminant can gain •[2] only. This is for a correct evaluation of the wrong expression but it must involve using a, b and c.
2. Candidates who clearly use $b = 4$ instead of $b = -4$ lose •[2]
3. •[3] is only available as a consequence of interpreting a numerical value for the discriminant.

Outcome 2

NB
- Throughout this Outcome treat the omission of dx as bad form.
- **In questions 4 and 5** candidates who attempt to find a solution via a graphics calculator earn no marks. The only acceptable solution is via calculus.

Qs	Give 1 mark for each •	Illustrations for awarding each mark
3	$\int \frac{7}{x^6}\,dx = -\frac{7}{5x^5} + c$	
	•1 Express in integrable form	•1 $7x^{-6}$
	•2 Integrate term with a negative power	•2 $\frac{7}{-5}x^{-5}$
	•3 Constant of integration	•3 $+c$
Notes: 1 •3 is only available if a candidate makes an attempt to integrate, even if it is a very crude attempt, ie any expression other than the original with $+c$ can be awarded •3. 2 •1 is the only mark available to candidates who use differentiation.		
4	6 square units	
	•1 Know to integrate	•1 $\int \ldots$
	•2 Write in integrable form and state limits	•2 $\int_0^2 x^3 - 3x^2 + 5x\,dx$
	•3 Integrate	•3 $\frac{1}{4}x^4 - x^3 + \frac{5}{2}x^2$
	•4 Substitute limits	•4 $(\frac{1}{4}(2)^4 - (2)^3 + \frac{5}{2}(2)^2) - (\frac{1}{4}(0)^3 - (0)^2 + \frac{5}{2}(0)^2)$
	•5 Process limits	•5 6

Notes:

1 The appearance of $\int$ and nothing else does not gain •1.

2 Candidates who write $\int_0^2 x(x^2 - 3x + 5)\,dx$ and no more can only gain •1

3 •4 and •5 are not available for substitution into original integrand.

4 Differentiation loses •3, •4 and •5.

5 Since the area is totally above the x-axis, •5 is not available for a negative answer irrespective of whether or not the candidate tries to deal with it.

Outcome 2 (continued)

Qs	Give 1 mark for each •	Illustrations for awarding each mark
5	$\frac{32}{3}$ or $10\frac{2}{3}$ or $10 \cdot 66...$ square units	
	•¹ Know to integrate and state limits	•¹ $\int_0^4 \cdots$
	•² Use 'upper – lower'	•² $\int \text{"}upper - lower\text{"}$ **stated or implied by •³**
	•³ Interpret upper – lower	•³ $4x - x^2$
	•⁴ Integrate	•⁴ $2x^2 - \frac{1}{3}x^3$
	•⁵ Substitute limits	•⁵ $(2(4)^2 - \frac{1}{3}(4)^3) - (2(0)^2 - \frac{1}{3}(0)^3)$
	•⁶ Process limits	•⁶ $\frac{32}{3}$ or equivalent

Notes:

1. Do not penalise candidates who work with $3 + 5x - x^2 - x - 3$ throughout in an unsimplified form.
2. Candidates who use $3 + 5x - x^2 - x + 3$ leading to $6 + 4x - x^2$ lose •³ but all other marks are still available.
3. Differentiation loses •⁴, •⁵ and •⁶.
4. •² is lost for subtracting the wrong way round and subsequently •⁶ may be lost for statements such as :
   $-\frac{32}{3}$ or $-\frac{32}{3}$ sq units or $-\frac{32}{3} = \frac{32}{3}$ or $-\frac{32}{3}$ so ignore negative

   •⁶ may be gained for statements such as $-\frac{32}{3}$ so area is $\frac{32}{3}$
5. $\int_4^0 \text{"}lower - upper\text{"}$ is correct and so all six marks are still available.

Outcome 3

Qs	Give 1 mark for each •	Illustrations for awarding each mark
6	$\frac{\pi}{12}$ and $\frac{5\pi}{12}$ or decimal equivalent $(0 \cdot 2617993\cdots$ **and** $1 \cdot 308996\cdots)$	
	•[1] Solve equation for $2x$ •[2] Process solutions for x	•[1] $2x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$ •[2] $x = \frac{\pi}{12}$ and $\frac{5\pi}{12}$
Notes:	1 Accept solutions as decimals to at least 2 decimal places. 2 Candidates who work in degrees throughout the question and convert their solutions to radians may be awarded full marks. 3 Candidates who work in degrees throughout the question and do not convert their solutions to radians lose •[2]. 4 As shown in the marking scheme (•[1] and •[2]) can be marked horizontally or vertically.	
7(a)	$\cos x = \frac{15}{17}$ and $\cos y = \frac{6}{10}$	
	•[1] Interpret diagram for $\cos x$ •[2] Interpret diagram for $\cos y$	•[1] $\cos x = \frac{15}{17}$ •[2] $\cos y = \frac{6}{10}$
7(b)	$\frac{77}{85}$	
	•[3] Use compound angle formula •[4] Substitute and complete	•[3] $\cos(x-y) = \cos x \cos y + \sin x \sin y$ •[4] $\frac{15}{17} \times \frac{6}{10} + \frac{8}{17} \times \frac{8}{10} = \frac{154}{170} = \frac{77}{85}$
Notes:	In (a) 1 The evidence for •[1] and •[2] may not be evident until (b). In (b) 2 •[3] may be stated or implied by •[4] 3 Simply stating the formula for $\cos(A-B)$ with no further working gains no marks. 4 Calculating approximate angles using arcsin ($\sin^{-1}$) and arccos ($\cos^{-1}$) gains no credit.	

Outcome 3 (continued)

Qs	Give 1 mark for each •	Illustrations for awarding each mark
8(a)	$\sin(x-31)^o$	
	•[1] Express in form $\sin(x-a)^o$	•[1] $\sin(x-31)^o$
8(b)	$77\cdot7$ and $164\cdot3$	
	•[2] Know to express in standard form	•[2] $\sin(x-31)^o = \dfrac{8}{11}$ **stated explicitly**
	•[3] Solve equation for one value of $x-31$	•[3] $46\cdot7$
	•[4] Process second solution	•[4] $133\cdot3$
	•[5] Process solutions for x.	•[5] $77\cdot7$ and $164\cdot3$

Notes: In (a) 1 Accept $a = 31$ for •[1]

2 Do not penalise the omission of the degree sign for •[1] or •[2].

In (b) 3 •[5] is for both answers and there is no horizontal or vertical marking here.

Outcome 4

Qs	Give 1 mark for each •	Illustrations for awarding each mark
9(a)	$(x+7)^2+(y+4)^2=36$	
	•[1] Interpret centre •[2] Interpret radius and complete equation	•[1] $(x-(-7))^2+(y-4)^2=...$ •[2] $...=36$
9(b)	Centre (2, –1) Radius $\sqrt{12}$ units	
	•[3] State centre of circle •[4] Know how to and find radius of circle	•[3] (2, –1) •[4] $r=\sqrt{2^2+(-1)^2-(-7)}=\sqrt{12}$ or equivalent
Notes:	1 In (a) •[2] is not awarded for 6^2, this must be simplified to 36. 2 In (b) $\sqrt{12}$ does not need to be simplified for •[2]. Also accept decimal equivalent 3·46.	
10	Line is a tangent to circle	
	•[1] Substitute eqn of line into eqn of circle •[2] Expand brackets •[3] Express in standard form •[4] Start test for tangency •[5] Complete test and communicate result	•[1] $x^2+(3x-2)^2-8x+6=0$ •[2] $x^2+9x^2-12x+4-8x+6=0$ •[3] $10x^2-20x+10=0$ •[4] $10(x-1)^2=0$ **or** $b^2-4ac=(-20)^2-4\times10\times10=400-400=0$ •[5] equal roots $\Rightarrow$ line is a tangent **or** $b^2-4ac=0\Rightarrow$ line is a tangent
Notes:	1 An "$=0$" must appear somewhere in the working between •[1] and •[4] stage. Failure to appear will lose one of these marks. 2 For candidates who obtain 2 roots; •[5] is still available for statements such as: 'not equal roots so not a tangent' **or** 'discriminant not 0 so not a tangent.	

Outcome 4 (continued)

Qs	Give 1 mark for each •	Illustrations for awarding each mark
11	$x - 3y + 16 = 0$	
	•[1] Find gradient of radius	•[1] $m_{RADIUS} = -3$
	•[2] State gradient of tangent	•[2] $m_{\text{TAN}GENT} = \frac{1}{3}$
	•[3] State equation of tangent	•[3] $y - 6 = \frac{1}{3}(x - 2)$