

Support Materials National Assessment Bank pack

Mathematics Higher

Mathematics 2 D322 12/NAB004

SCQF Level 6

Publication date: June 2010 Publication code: D322 12/NAB004

The information in this publication may be reproduced to support SQA qualifications. This publication must not be reproduced for commercial or trade purposes. **This material is for use by teaching staff only.**

Published by the Scottish Qualifications Authority, The Optima Building, 58 Robertson Street, Glasgow, G2 8DQ and Ironmills Road, Dalkeith, Midlothian, EH22 1LE.

© Scottish Qualifications Authority 2010

Contents

| Section 1 | Performance Criteria |
|-----------|--------------------------|
| Section 2 | Instrument of Assessment |
| Section 3 | Marking information |

Section 1 – Performance Criteria

Outcome 1

Use the factor/remainder theorem and apply quadratic theory.

Performance Criteria

- (a) Apply the factor/remainder theorem to a polynomial function.
- (b) Determine the nature of the roots of a quadratic equation using the discriminant.

Outcome 2

Use basic integration.

Performance Criteria

- (a) Integrate functions reducible to sums of powers of *x* (definite and indefinite).
- (b) Find the area between a curve and *x*-axis using integration.
- (c) Find the area between two curves using integration.

Outcome 3

Solve trigonometric equations and apply trigonometric formulae.

Performance Criteria

- (a) Solve a trigonometric equation in a given interval.
- (b) Apply trigonometric formulae (addition formulae) in the solution of a geometric problem.
- (c) Solve a trigonometric equation involving an addition formula in a given interval.

Use the equation of the circle.

Performance Criteria

- (a) Given the centre (a,b) and radius r, find the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$.
- (b) Find the radius and centre of a circle given the equation in the form $x^2 + y^2 + 2gx + 2fy + c = 0$.
- (c) Determine whether a given line is a tangent to a given circle.
- (d) Determine the equation of the tangent to a given circle given the point of contact.

Section 2 – Instrument of Assessment

Formulae list

Circle The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre (*-g,-f*) and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a,b) and radius *r*.

Trigonometric formulae $sin(A \pm B) = sin A cos B \pm cos A sin B$ $cos(A \pm B) = cos A cos B \mp sin A sin B$ sin 2A = 2 sin A cos A $cos 2A = cos^2 A - sin^2 A$ $= 2 cos^2 A - 1$ $= 1 - 2 sin^2 A$

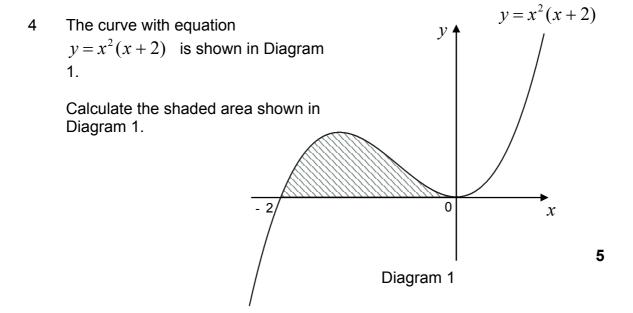
Unit assessment – Mathematics 2 (Higher)

Outcome 1

- 1 (i) Show that (x-1) is a factor of $f(x) = x^3 3x^2 x + 3$.
 - (ii) Hence factorise f(x) fully.
- 2 Determine the nature of the roots of the equation $4x^2 2x 3 = 0$ using the discriminant.

Outcome 2

3 Find
$$\int \frac{13}{x^4} dx$$
, $x \neq 0$.



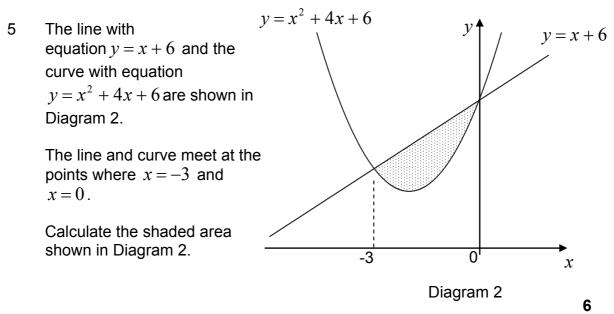
June 2010

6

3

3

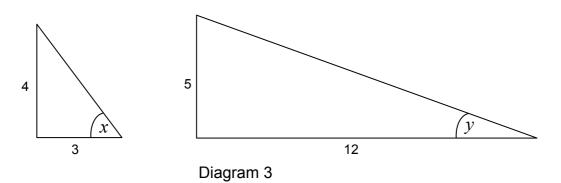
Marks



Outcome 3

6 Solve the equation
$$\sin 2x = \frac{\sqrt{3}}{2}$$
 for $0 < x < \pi$.

7 Diagram 3 shows two right-angled triangles.



(a) Write down the values of of $\sin x$ and $\sin y$. 2

(b) Show that the exact value of
$$sin(x - y)$$
 is $\frac{33}{65}$.

Marks

2

2

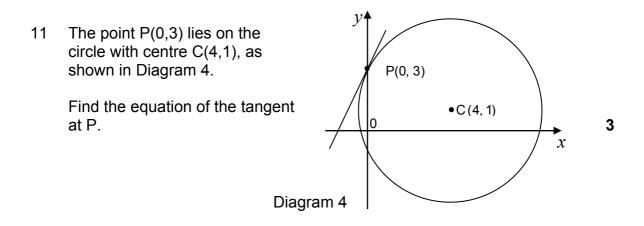
5

8 (a) Express
$$\cos x^{\circ} \cos 15^{\circ} + \sin x^{\circ} \sin 15^{\circ}$$
 in the form $\cos(x-a)^{\circ}$. 1

(b) Using the result from (a),
solve
$$\cos x^o \cos 15^o + \sin x^o \sin 15^o = \frac{7}{9}$$
 for $0 < x < 360$.

Outcome 4

- 9 (a) A circle has radius 3 units and centre (-2,5). Write down the equation of the circle.
 - (b) A circle has equation $x^2 + y^2 + 2x 4y 3 = 0$. Write down the coordinates of its centre and the length of its radius.
- 10 Show that the line with equation y = 3x 8 is a tangent to the circle with equation $x^2 + y^2 12x + 26 = 0$.



End of assessment

Section 3 – Marking information

Test specification grid

The grid below shows how the Outcomes and Performance Criteria are assessed in this Unit assessment.

| Торіс | PC | Question | Marks | Total | Threshold |
|---------------------------------|------|----------|-------|-------|-----------|
| Factor/remainder | 1(a) | 1 | 5 | 0 | - |
| theorem and quadratic theory | 1(b) | 2 | 3 | 8 | 5 |
| | 2(a) | 3 | 3 | | |
| Regio interretion | | 4 | | 4.4 | 10 |
| Basic integration | 2(b) | 4 | 5 | 14 | 10 |
| | 2(c) | 5 | 6 | | |
| Trigonometric | 3(a) | 6 | 2 | | |
| equations and | 3(b) | 7 | 4 | 11 | 7 |
| formulae | 3(c) | 8 | 5 | | |
| | 4(a) | 9 | 2 | | |
| Equation of the | 4(b) | 9 | 2 | 12 | 8 |
| circle | 4(c) | 10 | 5 | 12 | 0 |
| | 4(d) | 11 | 3 | | |

Marking information – Mathematics 2 (Higher)

Recommended general marking information

General marking instructions

- 1 Marks should be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
- 2 Award one mark for each (bullet point). Each error should be underlined at the point in the working where it first occurs, and not any subsequent stage of the working.
- 3 The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided the level of difficulty is approximately similar. Where, subsequent to an error, the working is eased, a deduction of marks(s) should be made.
- 4 As indicated on the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking information.
- 5 Do not penalise:
 - working subsequent to a correct answer
 - omission of units (except where marks are awarded for this in the detailed marking instructions)
 - legitimate variations in numerical answers
 - correct working in the wrong part of a question
 - bad form
- 6 No piece of work should be scored through without careful checking even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking information – answers which are widely off beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for marks.
- 7 No marks should be deducted for careless or badly arranged work.
- 8 Transcription errors In general, as a consequence of a transcription error, candidates lose the opportunity of gaining the first accuracy or processing mark.

- 9 Casual errors In general, as a consequence of a casual error, candidates lose the opportunity of gaining the first accuracy or processing mark.
- 10 Acceptable alternative methods of solution can only be given the marks specified in the marking information if the question does not stipulate the method candidates are to use to find the solution. In such circumstances, no marks may be awarded even though the candidate may have obtained the correct answer.
- 11 In general do not penalise the same error twice in the one question.
- 12 If an answer is scored out and not replaced, the scored out working should be marked where it is legible.
- 13 If a candidate presents more than one complete solution to a question and it is not clear which is intended as their final attempt, then each attempt should be marked and the lowest mark awarded. It is anticipated that this will be a rare occurrence.

Marking signs and abbreviations

 \checkmark

It is recommended that markers use the following signs and abbreviations for marking purposes:

Tick when a piece of working is correct and gains a mark.

| ≯ | A cross-tick should be used to indicate 'correct working' where a mark is awarded as a result of follow through from an error. |
|-----------|---|
| * | A double cross-tick should be used to indicate correct working which is inadequate to score any marks eg incorrect method which is mathematically correct or eased working. |
| × | Underline and cross each error especially those where a mark has been lost. |
| \sim | A tilde should be used to indicate a minor transgression which is not being penalised, eg bad form. |
| ^ | Use a roof to show that something is missing such as a crucial step in the working or part of a solution. |
| В | An upper case B should be used to indicate that you have given the candidate the benefit of the doubt and awarded a mark. |
| Е | An upper case E should be used to indicate that the candidate has eased the working as a consequence of an error and that marks have been deducted as a result. |
| Note – In | Course assessments, the letters B and E would not be used. |
| | |

| Qs | Give 1 mark for each • | Illustrations for awarding each mark | | |
|-----|---|--|--|--|
| 1 | f(x) = (x-1)(x+1)(x-3) | _ | | |
| | | Method 1 | | |
| | • Know to use $x = 1$ | •1 know to use $x = 1$ | | |
| | • ² Complete evaluation and conclusion | • ² $1-3-1+3=0 \Longrightarrow x-1$ is a factor | | |
| | • ³ Start to find quadratic factor | • ³ $(x-1)(x^2)$ | | |
| | • ⁴ Complete quadratic factor | • $(x-1)(x^2-2x-3)$ | | |
| | ● ⁵ Factorise completely | • ⁵ $(x-1)(x+2)(x-3)$ stated explicitly | | |
| | | OR | | |
| | | Method 2 | | |
| | | • 1 1 -3 -1 3 | | |
| | | • ² 1 1 -3 -1 3 1 -2 -3 | | |
| | | 1 -2 -3 0 | | |
| | | • ³ $f(1) = 0$ so $(x-1)$ is a factor | | |
| | | $\bullet^4 x^2 - 2x - 3$ | | |
| | | • ⁵ $(x-1)(x+1)(x-3)$ stated explicitly | | |
| Not | Notes | | | |
| 1 | In method 1, • ² and method 2, • ³ , candidates must show some acknowledgement of the resulting '0'. Do not accept anything simply as underlining or boxing in the zero. | | | |
| 23 | • ⁵ is for the product of the correct th In (ii) the correct answer only withou | | | |

3 In (ii) the correct answer only without working loses •³ and •⁴.

Outcome 1 (continued)

| Qs | | Give 1 mark for each • | Illustrations for awarding each mark |
|---|----------------|---|---|
| 2 | Two | distinct real roots | |
| | • ¹ | Know discriminant | • $b^2 - 4ac$ stated or implied by • ² |
| | | Substitute for <i>a</i> , <i>b</i> and <i>c</i> and evaluate | • ² $(-2)^2 - 4 \times 4 \times (-3) = 52$ |
| | • ³ | State nature of roots | • ³ Distinct (real) roots |
| Notes: 1 Any other expression masquerading as the discriminant can gain only. This is for a correct evaluation of the wrong expression but must involve using <i>a</i>, <i>b</i> and <i>c</i>. 2 Candidates who clearly use <i>b</i> = 2 instead of <i>b</i> = -2 lose •² •³ is only available as a consequence of interpreting a numerical v for the discriminant. 4 Do not award •³ for 'distinct roots', without the word 'real' appearing the construction of the word 'real' appearing the discriminant. | | aluation of the wrong expression but it c. $b = 2$ instead of $b = -2$ lose \bullet^2 sequence of interpreting a numerical value | |

- **NB** Throughout this Outcome treat the omission of dx as bad form.
 - In questions 4 and 5 candidates who attempt to find a solution via a graphics calculator earn no marks. The only acceptable solution is via calculus.

| Qs | Give 1 mark for each • | Illustrations for awarding each mark | |
|-----|--|--------------------------------------|--|
| 3 | $\int \frac{13}{x^4} dx = -\frac{13}{3x^3} + c$ | | |
| | •1 Express in integrable form | • $13x^{-4}$ | |
| | • ² Integrate term with a negative power | • ² $\frac{13}{-3}x^{-3}$ | |
| | • ³ Constant of integration | \bullet^3 + C | |
| Not | Notes: 1 •³ is only available if a candidate makes an attempt to integrate, even if it is a very crude attempt, ie any expression other than the original with + c can be awarded •³. 2 •¹ is the only mark available to candidates who use differentiation. | | |

Outcome 2 (continued)

| 4 | $\frac{4}{3}$ or $1\frac{1}{3}$ or 1.333 square units | | |
|-----|---|---|--|
| | • ¹ Know to integrate | • ¹ ∫ | |
| | • ² Write in integrable form and state limits | • ² $\int_{-2}^{0} x^3 + 2x^2 dx$ | |
| | • ³ Integrate | • ³ $\frac{1}{4}x^4 + \frac{2}{3}x^3$ | |
| | ⁴ Substitute limits | • $(\frac{1}{4}(0)^4 + \frac{2}{3}(0)^3) - (\frac{1}{4}(-2)^4 + \frac{2}{3}(-2)^3)$ | |
| | ●⁵ Process limits | • ⁵ $\frac{4}{3}$ or equivalent | |
| Not | Notes: | | |
| 1 | The appearance of \int and nothing else does not gain • ¹ . | | |
| 2 | Candidates who write $\int_{-2}^{0} x^2 (x+2) dx$ and no more can only gain \bullet^1 | | |
| 3 | \bullet^4 and \bullet^5 are not available for substitution into original integrand. | | |
| 4 | Differentiation loses \bullet^3 , \bullet^4 and \bullet^5 . | | |
| 5 | - | <i>x</i> -axis, • ^₅ is not available for a negative not the candidate tries to deal with it. | |

Outcome 2 (continued)

| Qs | Gi | ve 1 mark for each • | Illustrations for awarding each mark |
|-----|-----------------------|--|--|
| 5 | $\frac{9}{2}$ or 4 | $\frac{1}{2}$ 4 · 5 square units | |
| | limit | w to integrate and state is e 'upper – lower' | • $\int_{-3}^{0} \dots$ • $\int "upper - lower"$ stated or implied by • ³ |
| | • ³ Inte | rpret upper – lower | • $-3x - x^2$ • $-\frac{3}{2}x^2 - \frac{1}{3}x^3$ |
| | •⁴ Inte | grate | 2 3 3 ··· |
| | •⁵ Sub | ostitute limits | $\left(-\frac{3}{2}(0)^2 - \frac{1}{3}(0)^3\right) - \left(-\frac{3}{2}(-3)^2 - \frac{1}{3}(-3)^3\right)$ |
| | • ⁶ Pro | cess limits | • ⁶ $\frac{9}{2}$ or equivalent |
| Not | tes: 1 2 3 4 | throughout in an unsimp Candidates who use $x + \frac{1}{2}$ • ³ but all other marks a Differentiation loses • ⁴ , • ² is lost for subtracting may be lost for stateme $-\frac{9}{2}$ or $-\frac{9}{2}$ sq units • ⁶ may be gained for st | + $6 - x^2 + 4x + 6$ leading to $12 + 5x - x^2$ lose re still available. • ⁵ and • ⁶ . the wrong way round and subsequently • ⁶ nts such as : or $-\frac{9}{2} = \frac{9}{2}$ or $-\frac{9}{2}$ so ignore negative atements such as $-\frac{9}{2}$ so area is $\frac{9}{2}$ |
| | 5 | $\int_{0} iower - upper^{-1}$ is c | orrect and so all six marks are still available. |

| Qs | Give 1 mark for each • | Illustrations for awarding each mark | |
|-------|--|--|--|
| 6 | $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or decimal equivalent (0) | · 523598and 1·047197) | |
| | •¹ Solve equation for 2<i>x</i> •² Process solutions for <i>x</i> | • $2x = \frac{\pi}{3}$ and $\frac{2\pi}{3}$ • $x = \frac{\pi}{6}$ and $\frac{\pi}{3}$ | |
| Note | es: 1 Accept solutions as decima | ls to at least 2 decimal places. | |
| | 2 Candidates who work in de their solutions to radians m | grees throughout the question and convert ay be awarded full marks. | |
| | 3 Candidates who work in de convert their solutions to ra | grees throughout the question and do not dians lose • ² . | |
| | 4 As show in the marking sch or vertically. | neme (\bullet^1 and \bullet^2) can be marked horizontally | |
| 7(a) | $\sin x = \frac{4}{5} \text{ and } \sin y = \frac{5}{13}$ | | |
| | • ¹ Interpret diagram for $\sin x$ | •1 $\sin x = \frac{4}{5}$ | |
| | • ² Interpret diagram for $\sin y$ | $\bullet^2 \sin y = \frac{5}{13}$ | |
| 7(b) | 33 | | |
| | 65 | | |
| | • ³ Use compound angle formula | • ³ $\sin(x-y) = \sin x \cos y - \cos x \sin y$ | |
| | ⁴ Substitute and complete | •4 $\frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} = \frac{33}{65}$ | |
| Note | ès: | | |
| In (a | | | |
| In (b | | - | |
| | 3 Simply stating the formula for $sin(A - B)$ with no further working gains no marks. | | |
| | 4 Calculating approximate ang gains no credit. | les using arcsin (sin ⁻¹) and arccos (cos ⁻¹) | |

Outcome 3 (continued)

| Qs | Give 1 mark for each • | Illustrations for awarding each mark |
|------|--|--|
| 8(a) | $\cos(x-15)^{\circ}$ | |
| | •1 Express in form $\cos(x+a)^{\circ}$ | • $\cos(x-15)^{\circ}$ |
| 8(b) | $53 \cdot 9$ and $336 \cdot 1$ | |
| | • ² Know to express in standard f | Form $e^2 \cos(x-15)^\circ = \frac{7}{9}$ stated |
| | | explicitly |
| | • Solve equation for one value of $x - 15$ | of • ³ 38.9 |
| | • ⁴ Process second solution | • ⁴ 321·1 |
| | • 5 Process solutions for <i>x</i> . | • ⁵ 53 ⋅ 9 and 336 ⋅ 1 |
| Note | s: In (a) 1 Accept $a = 15$ for • ¹ | |
| | 2 Do not penalise the o | pmission of the degree sign for \bullet^1 or \bullet^2 . |
| | In (b) 3 • ^₅ is for both answer marking here. | s and there is no horizontal or vertical |
| | | |

| Qs | Give 1 mark for each • | Illustrations for awarding each mark | |
|------|---|---|--|
| 9(a) | $(x+2)^2 + (y-5)^2 = 9$ | | |
| | •1 Interpret centre | • $(x - (-2))^2 + (y - 5)^2 =$ | |
| | Interpret radius and complete equation | • ² = 9 | |
| 9(b) | Centre (–1, 2) Radius $\sqrt{8}$ units | | |
| | • ³ State centre of circle | • ³ (-1, 2) | |
| | •⁴ Know how to and find radius of circle | •4 $r = \sqrt{1}^{2} + (-2^{2} - (-3)) = \sqrt{8}$ | |
| Note | s: 1 In (a) \bullet^2 is not awarded for | 3^2 , this must be simplified to 9. | |
| | 2 In (b) $\sqrt{8}$ does not need to equivalent 2.83. | be simplified for • ² . Also accept decimal | |
| 10 | Line is a tangent to circle | | |
| | Substitute eqn of line into eqn of circle | • $x^{2} + (3x-8)^{2} - 12x + 26 = 0$ | |
| | • ² Expand brackets | • ² $x^2 + 9x^2 - 48x + 64 - 12x + 26 = 0$ | |
| | • ³ Express in standard form | • $10x^2 - 60x + 90 = 0$ | |
| | • ⁴ Start test for tangency | • $10(x-3)^2 = 0$ | |
| | | or $b^2 - 4ac = (-60)^2 - 4 \times 10 \times 90$ | |
| | | = 3600 - 3600 = 0 | |
| | Complete test and communicate result | •⁵ equal roots ⇒ line is a tangent or | |
| | | $b^2 - 4ac = 0 \Longrightarrow$ line is a tangent | |
| Note | s: 1 An "= 0" must appear som stage. Failure to appear with the stage of the stage | ewhere in the working between •1 and •4 ill lose one of these marks. | |
| | For candidates who obtain 2 roots; •⁵ is still available for statements such as: | | |
| | 'not equal roots so no | t a tangent' | |
| | or 'discriminant not 0 so | not a tangent. | |

Outcome 4 (continued)

| Qs | Give 1 mark for each • | Illustrations for awarding each mark |
|----|--|---|
| 11 | 2x - y + 3 = 0 | |
| | •1 Find gradient of radius | • $m_{RADIUS} = -\frac{1}{2}$ |
| | • ² State gradient of tangent | • ² $m_{\text{TANGENT}} = 2$ |
| | • ³ State equation of tangent | • $y - 3 = 2(x - 0)$ |