Higher Maths – Homework 10

Non-calculator section:

1. tan x = ½. The value of sin2x is

A $\frac{2}{5}$ B $\frac{2}{\sqrt{5}}$ C $\frac{4}{5}$ D $\frac{4}{\sqrt{5}}$

2. $f(x) = \frac{x+1}{x} - 1$ and $g(x) = \frac{2}{x}$ $x \neq 0$. f(g(x)) could be written as

A x B 2x C $\frac{1}{2}x$ D None of these

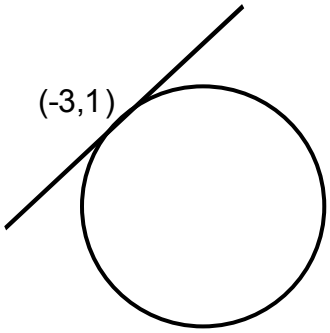
3. When $2 - 4x - x^2$ is written in the form $a - (x + b)^2$, the value of a is

A -2 B 6 C 2 D 4

4. (a) Given (x + 2) is a factor of $f(x) = x^3 - px - 6$, find p.
   (b) Hence factorise f(x) fully.

5. (a) A is the point (3,1,-4) and C is (15,13,-16). B divides AC in the ratio 2:1, find the coordinates of B.
   (b) D is the point (12,11,-10) and E is (8,3,-18). Show that D, B and E are collinear.

6. A circle has equation $x^2 + y^2 - 6x + 8y - 36 = 0$. Find the equation of the tangent to this circle at the point (-3,1).



7. A curve has equation $y = x^2 - 12\sqrt{x}$.
   Find the equation of the tangent to this curve at the point where x = 4.

8. PQR is a triangle with P(2,-1), Q(2,9) and R(5,0).

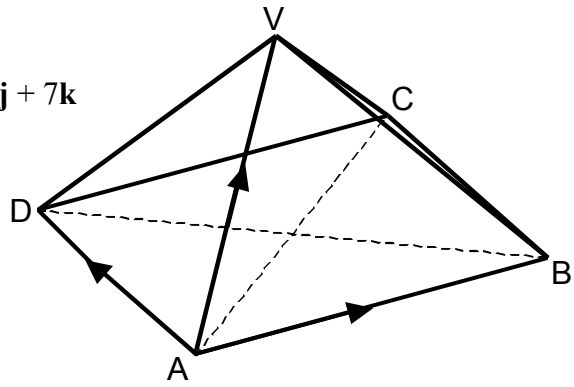
   (a) Find the equation of the perpendicular bisector of PQ.
   (b) Find the equation of the altitude from P to QR.
   (c) Find the point of intersection of these two lines.

Calculator section:

9. $f'(x) = 6(2x - 1)^2$ and f(2) = 17.
   Find a formula for f(x).

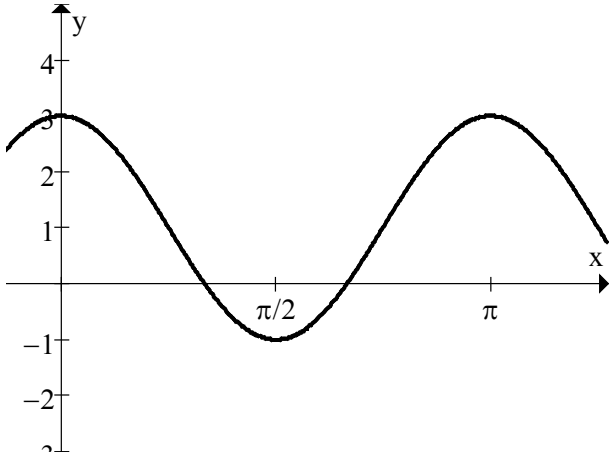
10. VABCD is a pyramid with rectangular base ABCD.
The vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AV}$ are given by

$\overrightarrow{AB} = 8\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ $\overrightarrow{AD} = -2\mathbf{i} + 10\mathbf{j} - 2\mathbf{k}$ $\overrightarrow{AV} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$

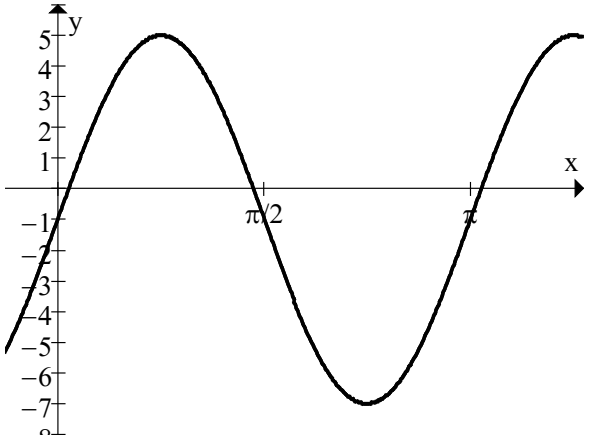


(a) Express $\overrightarrow{VC}$ in component form.
(b) Calculate the size of angle AVC.

11. (a) The diagram opposite shows the graph of
$f(x) = a\cos bx + c$.
Write down a formula for f(x).



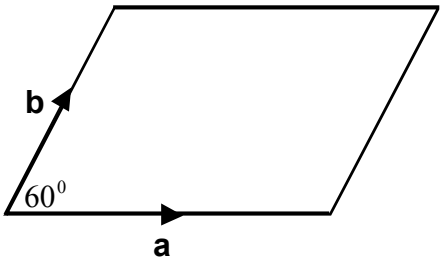
(b) The diagram opposite shows the graph of
$g(x) = a\sin bx + c$.
Write down a formula for g(x).



(c) Express f(x) + g(x) in the form $k\cos(2x - \alpha)$.

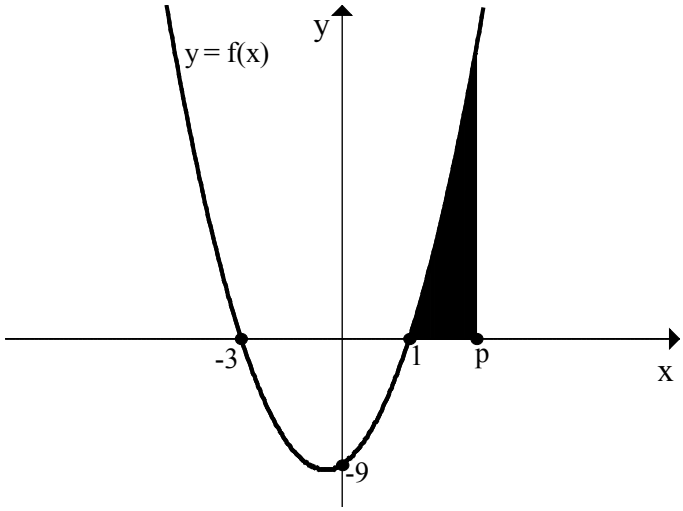
(d) Hence solve $f(x) + g(x) = \sqrt{10}$, $0 \le x \le 360$

12. The diagram opposite shows a parallelogram.
Given $|\mathbf{a}| = 5$ and $|\mathbf{b}| = 4$, show that $\mathbf{a}.(\mathbf{a} + \mathbf{b}) = 35$.



13. (a) The diagram shows the graph of $y = f(x)$.
Find a formula for $f(x)$.

(b) The shaded region has an area of 32 units.
Find p.



14. (a) State the condition for a quadratic equation to have equal roots.

(b) The equation $(x + k)^2 = k(x - 1) + 1$ has equal roots, find k.