Higher Maths – Homework 11

1. $f(x) = x^2 - 6x + 1$ is written in the form $y = (x + a)^2 + b$.
   The values of a and b are

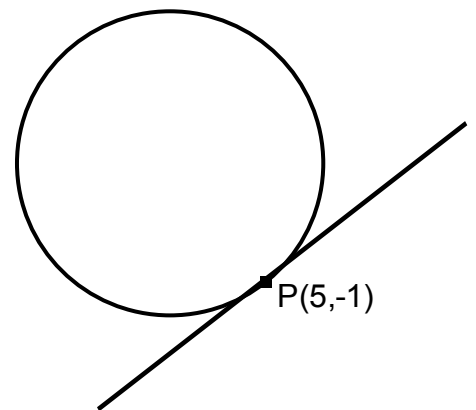   A $a = 3, b = 8$ B $a = -3, b = 8$ C $a = -3, b = -8$ D $a = 3, b = -8$

2. If $\sin x = \frac{3}{5}$, the value of $\cos 2x$ is

   A $\dfrac{7}{25}$ B $\dfrac{8}{5}$ C $\dfrac{25}{7}$ D $\dfrac{5}{8}$

3. $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix}$. The value of $(\mathbf{u} + \mathbf{v}).(\mathbf{u} - \mathbf{v})$ is

   A   -3   B   -1   C   11   D   3

4. A circle has equation $x^2 + y^2 - 6x + 4 = 0$.
   Find the equation of the tangent to this circle
   At the point P(5,-1).

P(5,-1)

5. Solve, for $x > 0$

$$\log_2 (x^2 - 9) - 2\log_2 (x - 3) = 2$$

6. Given $f'(x) = \dfrac{4}{\sqrt{2x - 1}}$ and $f(5) = 20$, find a formula for $f(x)$.

7. $f(x) = x^3 - 3x^2 - ax + b$.
   $(x + 2)$ and $(x - 3)$ are both factors of $f(x)$. Find the values of a and b.

8. $u_{n+1} = 0.6u_n + 10$ $u_2 = 52$.

   (a) Find u_0.
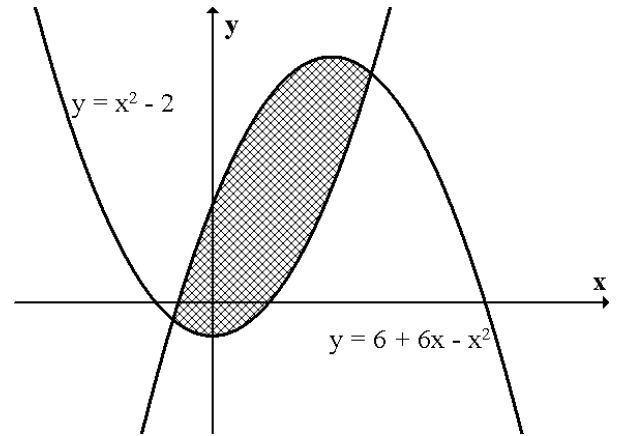   (b) Explain why this sequence has a limit and find the value of this limit.

9. $f(x) = \cos x - 7\sin x$.

    (a) Express $f(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 \le a \le 360$.

    (b) Hence, or otherwise, write down the maximum value of $f(x)$ and the value of
       x for which it occurs in the range $0 \le x \le 360$

10. The diagram opposite shows the graphs of

    $y = x^2 - 2$ and $y = 6 + 6x - x^2$.

    Calculate the shaded area.

11. Solve the equation $3\cos 2x = 5\cos x + 3$ $0 \le x \le 360$

12. A function is defined as $f(x) = 3\sin^2 x - \cos 2x$.
    Find $f'(\frac{\pi}{4})$.

13. (a) A is the point (2,-2,7) and B is (-8,3,-3). C divides AB in the ratio 2:3.
       Find the coordinates of C.

    (b) D is the point (-6,4,1). Find the coordinates of the point E given $\overrightarrow{CE} = \frac{3}{5}\overrightarrow{DE}$.

    (c) Calculate the size of angle ECB.

14. (a) A curve has equation $y = \dfrac{x^2}{4} - \dfrac{4}{x}$. Find the equation of the tangent to this
       curve at the point where $x = 2$.

    (b) Show that this tangent is also a tangent to the circle $x^2 + y^2 + 28x + 6y + 25 = 0$
       and find the point of contact.

15. The diagram shows the graph of $y = \log_3 x$.

    (a) Find the value of a

    (b) Sketch the graph of $y = \log_3 \dfrac{27}{x}$