Higher Maths – Homework 9

Non-calculator section:

1. A is the point (-3,0,-4) and B is (-1,-2,-5). The magnitude of the vector $\overrightarrow{AB}$ is

   A 9 B 3 C $\sqrt{3}$ D $\sqrt{101}$

2. The value of $2\tan^2\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right)$ is

   A 3 B $\frac{\sqrt{3}}{2}$ C $\frac{1}{3}$ D 1

3. The roots of the equation $kx^2 - 6x + 3k = 0$ are equal. Given $k > 0$, the value of k is

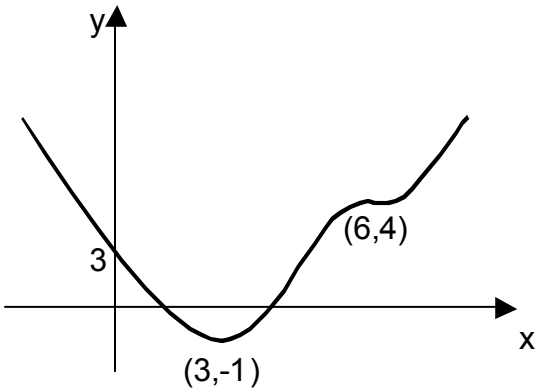
   A 36 B 3 C $\sqrt{6}$ D $\sqrt{3}$

4. Find the equation of the perpendicular bisector of the line joining the points A(-2,6) and B(4,4).

5. The curve with equation $y = x^3 - 6x^2 + 12x + 5$ has only one stationary point. Find this stationary point and determine its nature.

6. The graph of $y = f(x)$ is shown opposite.
   f(x) has turning points at (3,-1) and (6,4).

   Sketch the graph of $y = f'(x)$.



7. Two circles, A and B, have equations
   $(x + 2)^2 + (y - 4)^2 = 20$ and $x^2 + y^2 - 6x - 28y + 160 = 0$.

   (a) Show that the radius of circle B is $3\sqrt{5}$.
   (b) Show that circles A and B touch at a single point.
   (c) Find the coordinates of the point of contact.

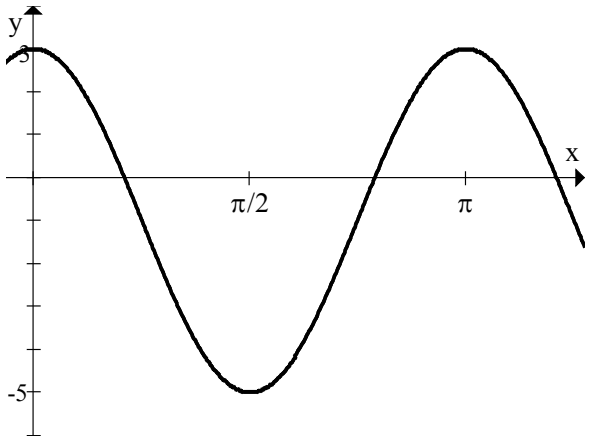
8. (a) A curve has equation $y = x^3 - 3x^2 + 4x + 2$. Find the equation of the tangent to this curve at the point where $x = 2$.
   (b) Find the coordinates of the point where this tangent meets the curve again.

Calculator section:

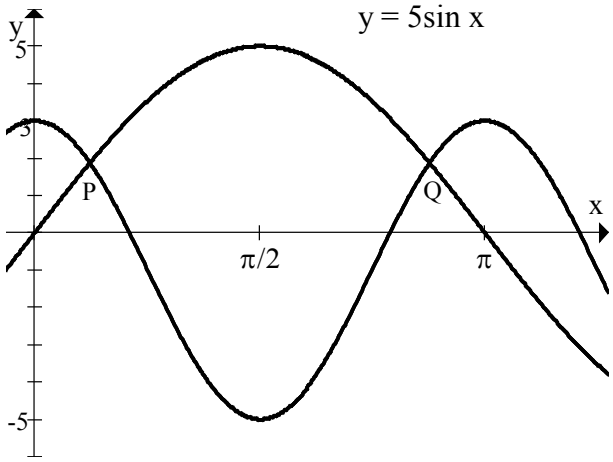
9. A recurrence relation is defined as $u_{n+1} = 0.4u_n + 10$, $u_o = 40$.

   (a) Find the smallest value of n for which $u_n < 17$.
   (b) Explain why this recurrence relation has a limit and find this limit.

10. (a) The diagram shows the graph of
$y = a\cos bx + c$.
Write down the values of a, b and c.

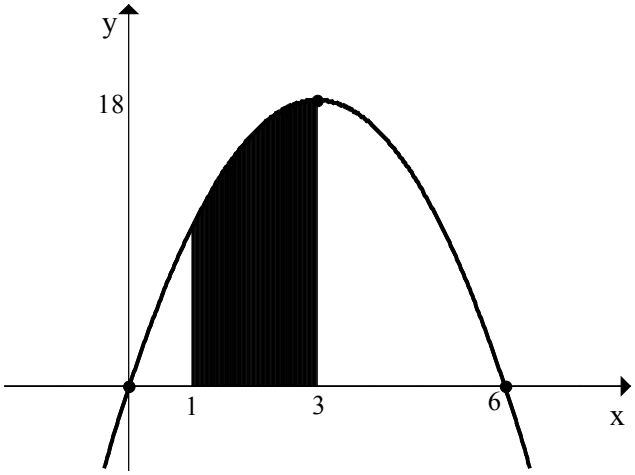


(c) Find the coordinates of P and Q, the points of intersection of the graph in (a) with the graph $y = 5\sin x$.



11. The diagram shows the graph of $y = f(x)$.

(a) Find a formula for f(x).
(b) Calculate the shaded area.



12. (a) Express $2\cos x + 4\sin x$ in the form $k\cos(x + \alpha)$ where $k > 0$ and $0 \leq \alpha \leq 360$.

(b) Write down the maximum value of $2\cos x + 4\sin x$ and the value of x at which this maximum occurs.