Higher_ Recurrence Relations.

1. The terms of a sequence satisfy $U_{n+1} = kU_n + 6$. Find the value of k which produces a sequence with a limit of 5. *(2)*

2. A sequence is defined by the recurrence relation
$U_{n+1} = \frac{1}{3} U_n + 9$, $U_o = 0$.
a) Calculate the values of U_1, U_2 and U_3. *(3)*

As $n \to \infty$ the sequence approachs a limit q.
b) Find the exact value of q *(2)*

3. A recurrence relation is defined by $U_{n+1} = pU_n + q$, where $-1 < p < 1$ and $U_o = 20$.

a) If $U_1 = 18$ and $U_2 = 17$, find the values of p and q. *(2)*

b) Find the limit of this recurrence relation as $n \to \infty$ *(2)*

4. Two equations are defined by the recurrence relations

$U_{n+1} = 0.2U_n + p$, $U_0 = 1$ and
$V_{n+1} = 0.6V_n + q$, $V_0 = 1$

a) Explain why each of these sequences has a limit. *(1)*

b) If both these sequences have the same limit, express p in terms of q. *(3)*

5. A doctor administers 20ml of a drug to a patient each day.
Over the same period it is estimated that 75% of the drug in the patient's bloodstream is removed.
If the level in the bloodstream rises above 30ml, the drug becomes toxic.

(a) Write down a recurrence relation that describes this situation. *(3)*

(b) Will it be safe to continue to administer this drug in the long term *(2)*

Total (20)