Higher Mathematics
Unit 2

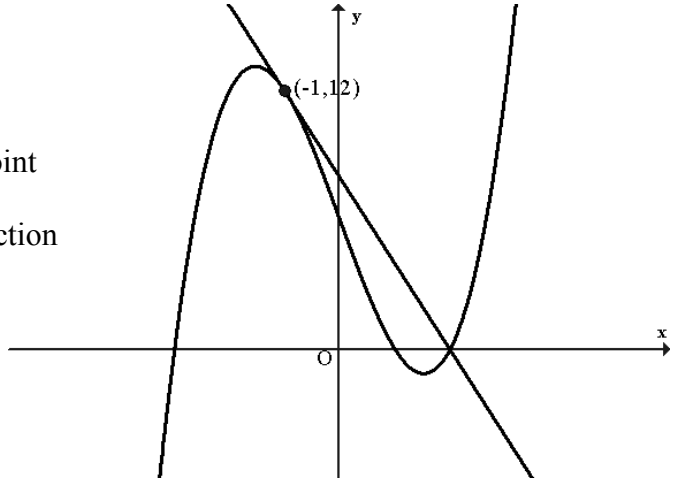
1. Show that $(x + 1)$ is a factor of $2x^3 + 5x^2 – 2x – 5$ and hence factorise $2x^3 + 5x^2 – 2x – 5$ completely.

2. Show that 4 is a root of $2x^3 –8x^2 – 8x + 32 = 0$ and hence find the other roots.

3. Given $(x – 2)$ is a factor of $f(x) = x^3 – x^2 + kx + 12$, find the value of k.
   Hence factorise f(x) completely.

4. (a) Given that $(x – 2)$ and $(x + 2)$ are both factors of $f(x) = x^3 + x^2 + px + q$, find the values of p and q.
   (b) Solve $f(x) = 0$ for these values.

5. The tangent to the curve $y = x^3 – 7x + 6$ at the point (-1,12) has equation $y + 4x = 8$.
   Find the coordinates of the other point of intersection of the curve and this tangent.



6. (a) Express in the form $f(x) = a(x + b)^2 + c$
   (b) Sketch the graph of each function clearly marking its turning point and where it crosses the y-axis.

   (i) $f(x) = x^2 – 6x + 15$ (ii) $f(x) = 10 – 8x – x^2$ (iii) $f(x) = 3x^2 + 12x – 1$

7. Show that the roots of $(t – 1)x^2 + 2tx + 4 = 0$ are real for all values of t.

8. The roots of $mx^2 + 4mx + 16 = 0$ are equal. Find the value of m given $m \neq 0$.

9. (a) Show that the equation $(x – 1)(x + k) = -4$ can be written in the form

$$x^2 + x(k – 1) + 4 – k = 0$$

   (b) The roots of the equation $(x – 1)(x + k) = -4$ are equal. Find the values of k.

10. A function has equation $f(x) = \frac{1}{2}x^4 + ax^2 + 24x - 1$.

(a) f(x) has a stationary point when x = -2. Find the value of a.

(b) Show that f(x) has no other stationary points.

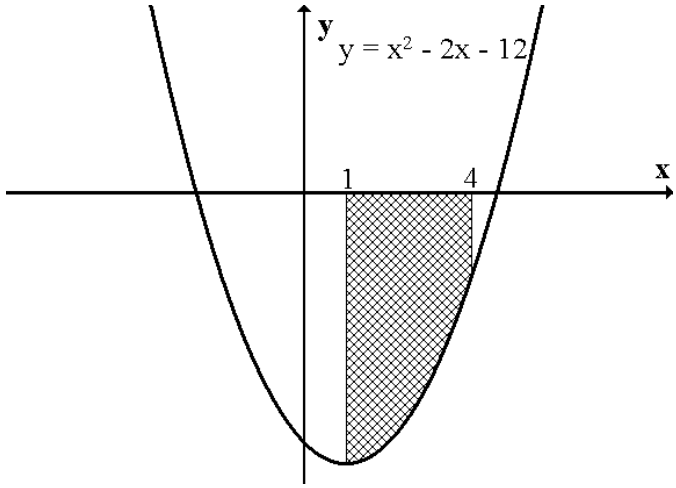
11. (a) Show that x = 2 is a solution to the equation $2x^3 + kx^2 - 2kx - 16 = 0$.
(b) Hence find the range of values of k for which all the roots of this equation are real.

12. $f'(x) = x^2 - 4x + 6$ and f(3) = 4. Find a formula for f(x).

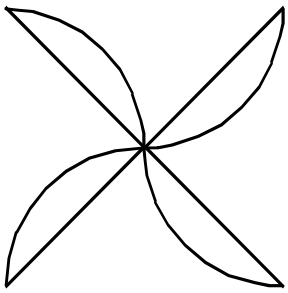
13. Given $\frac{dy}{dx} = 4x + 6\sqrt{x}$ and y = 50 when x = 4, find a formula for y.

14. The diagram shows the graph of $y = x^2 - 2x - 12$.

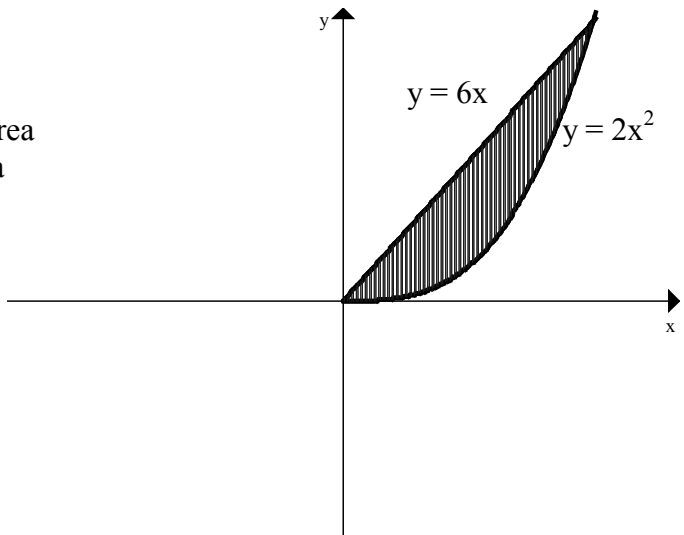
Calculate the shaded area.



15. The diagram opposite shows the design for the blades of a windmill.
All 4 blades are equal in size and are made from aluminium.



A single blade can be described as the area between the line y = 6x and the parabola $y = 2x^2$, as shown.
On the diagram each square unit represents $3m^2$

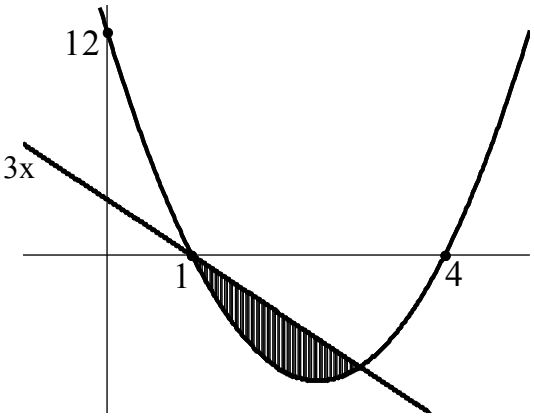


Calculate the total area of aluminium needed to make the blades.

16. The diagram opposite shows the line $y = 3 - 3x$ and the parabola f(x).

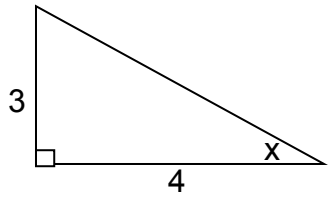
(a) Find a formula for f(x).

(b) Calculate the shaded area.



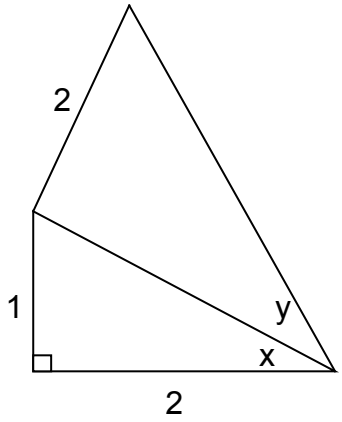
17. Given $\tan x = \frac{3}{4}$, find the exact value of

(a) cos 2x
(b) cos 4x



18. Using the information opposite show that

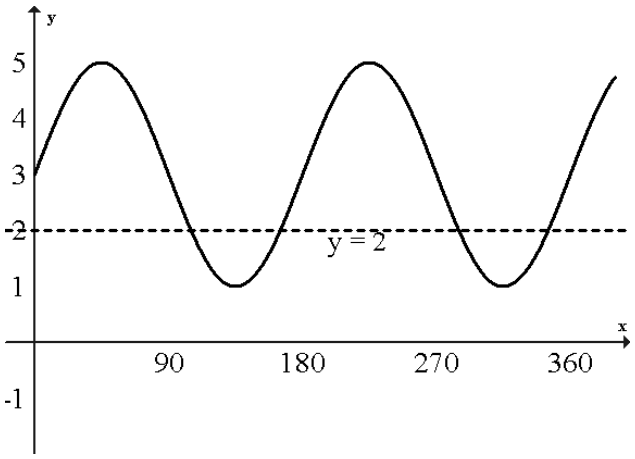
the exact value of $\cos(x + y)$ is $\frac{2\sqrt{5} - 2}{3\sqrt{5}}$



19. Solve the equations (a) $3\sin 2x = 3\cos x$ for $0 \le x \le 360$
(b) $2\cos 2x - 3\cos x + 1 = 0$ for $0 \le x \le 360$

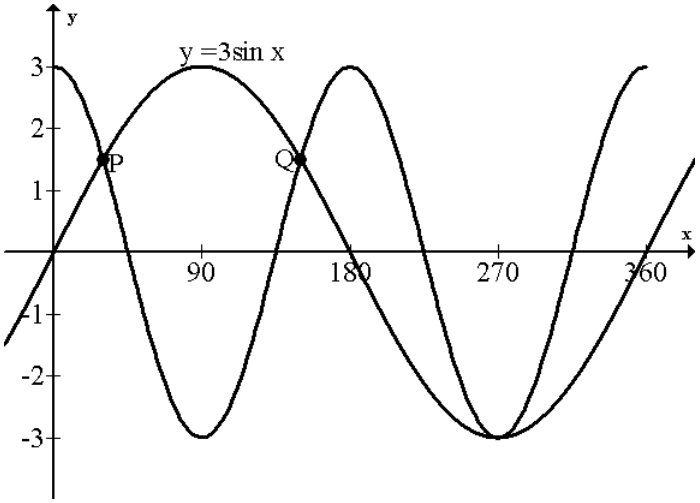
20. The diagram opposite shows the graph $y = a\sin bx + c$.

(a) Write down the values of a, b and c.
(b) Find the points of intersection between this curve and the line $y = 2$ for $0 \le x \le 360$

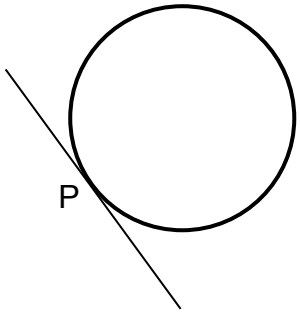


21. The diagram opposite shows the graphs of $y = a\cos bx$ and $y = 3\sin x$.

(a) Write down the values of a and b.
(b) Find the coordinates of P and Q.

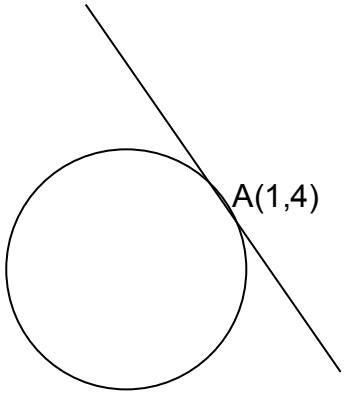


22. Find the equation of the tangent to the circle $x^2 + y^2 - 8x + 4y - 33 = 0$ at the point P(1,-4).



23. (a) Find the equation of the tangent to the circle $x^2 + y^2 + 10x - 2y - 19 = 0$ at the point A(1,4).

(b) Show that this tangent is also a tangent to the parabola $y = 2x^2 - 10x + 14$ and find the point of contact.



24. (a) A circle has centre (6,5) and radius $\sqrt{17}$. Show that the equation of this circle can be written in the form

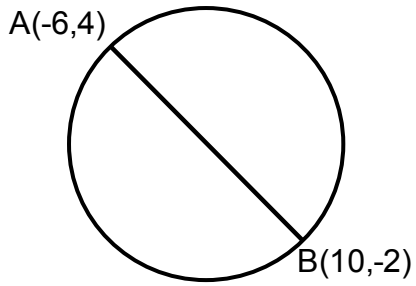
$$x^2 + y^2 - 12x - 10y + 44 = 0$$

(b) Show that the line $y = 4x - 2$ is a tangent to this circle and find the point of contact.

25. (a) A circle has centre (a,0) and radius 3. Write down the equation of this circle.

(b) The line $y = x$ is a tangent to this circle. Show that the exact value of a is $\pm 3\sqrt{2}$

26. A is the point (-6,4) and B is (10,-2). Find the equation of the circle which has AB as a diameter.



27. Two circles have equations

$$x^2 + y^2 + 4x + 16y - 60 = 0 \quad \text{and} \quad x^2 + y^2 - 8x + 4y + 12 = 0$$

Show that these circles touch at a single point.

28. A circle, centre Q, has equation $x^2 + y^2 - 2y - 1 = 0$.

(a) Find the equation of the tangent to this circle at the point P(1,2).

(a) There are two tangents to the circle which are parallel to the radius PQ.
Find the equations of these tangents.