Higher Mathematics
Unit 3

1. (a) Find the magnitude of the vector $\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix}$.

   (b) Find a vector parallel to the vector $\begin{pmatrix} -6 \\ 0 \\ 8 \end{pmatrix}$ which has unit length.

2. $\mathbf{u} = 3\mathbf{i} - 3\mathbf{j} + \sqrt{7}\,\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + a\mathbf{j} + \sqrt{8}\,\mathbf{k}$. If $|\mathbf{u}| = |\mathbf{v}|$ find the value of a.

3. (a) P has coordinates (2,-1,4) and R has coordinates (7,4,-1). Q divides PR in the ratio 3:2. Find the coordinates of Q.

   (b) T is (-1,0,-3) and U is (-10,-3,-9). Show that Q,T and U are collinear, stating the ratio of QT:TU.

4. (a) P is the point (1,2,-4) and Q is the point (-4,2,6). A divides PQ in the ratio 2:3. Find the coordinates of A.

   (b) B is the point (u,-1,-1) and C is (11,-7,-3). Given A, B and C are collinear, Find u.

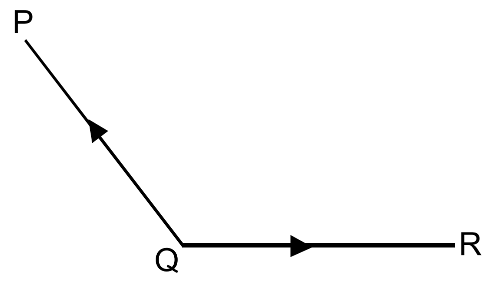
5. Show that the vectors $\mathbf{a} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = -6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ are perpendicular.

6. $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$.

   (a) Find the vectors $2\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$
   (b) Show that the vectors $2\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are perpendicular.

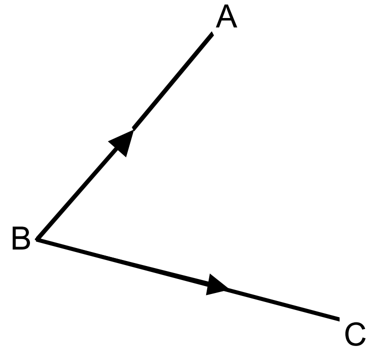
7. P has coordinates (1,-1,-1), Q is (3,0,1) and R is (7,4,3).

   Calculate the size of angle PQR.



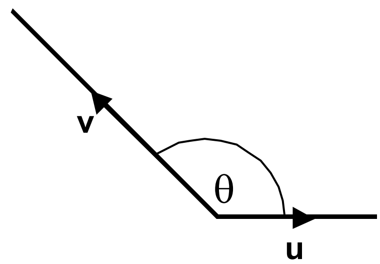
8. A is the point (-1,2,4), B is (0,4,2) and C 1s (-4,0,2).

Calculate the size of angle ABC.



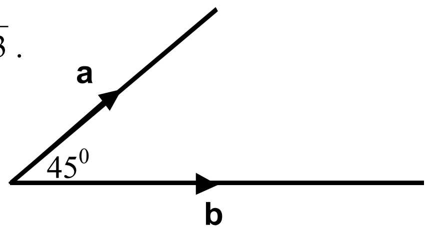
9. Two vectors **u** and **v** are such that $|\quad| = 2$ and $|\mathbf{v}| = 6$.

Given that $2\mathbf{u}.(\mathbf{u} + \mathbf{v}) = -4$, show that angle $\theta = 120^0$.



10. The diagram shows two vectors **a** and **b** with $|\quad| = 2$ and $|\mathbf{b}| = 3\sqrt{3}$.

(a) Evaluate (a) **a.a** (b) **b.b** (c) **a.b**
(b) Given **p** = 2**a** + 3**b** evaluate **p.p**.



11. Find the derivative of

(a) $y = (6x - 1)^4$ (b) $f(x) = 4\sqrt{3x - 1}$ (c) $y = \dfrac{6}{(2x-5)^2}$

(d) $f(x) = 4\cos 3x$ (e) $f(x) = 3\sin^2 x$

12. A curve has equation $y = (3x + 2)^4$. Find the equation of the tangent to this curve at the point where x = -1.

13. Show that the tangent to the curve $y = 4\sin(3x - \frac{\pi}{3})$ at the point where $x = \frac{\pi}{6}$ has equation $y - 2 = 6\sqrt{3}(x - \frac{\pi}{6})$.

14. Find the values of x for which the function $f(x) = \frac{1}{6}(2x-3)^3 - x$ is increasing.

15. (a) $\int (3x-4)^3\,dx$ (b) $\int \sqrt{4x+3}\,dx$ (c) $\int \dfrac{6}{(1-2x)^2}\,dx$ (d) $\int \sin(6x-2)\,dx$

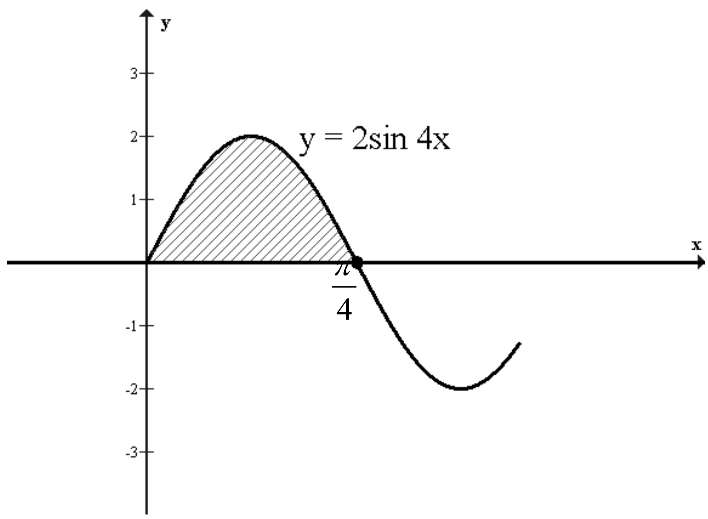
16. Evaluate $\int_1^2 \frac{8}{(4x - 1)^3} dx$

17. $\frac{dy}{dx} = \frac{1}{\sqrt{4x - 3}}$ and the curve passes through the point (1,2). Find a formula for y.

18. $\frac{dy}{dx} = 8\cos 4x$. This curve passes through the point $\left(\frac{\pi}{6}, 6\right)$. Find y.

19. The diagram shows part of the graph of $y = 2\sin 4x$.

Calculate the shaded area.



20. (a) Express $\sqrt{5}\cos x + 2\sin x$ in the form $k\cos(x - a)$ where $k > 0$ and $0 \leq a \leq 360$

(b) Hence write down the maximum value of $2 + \sqrt{5}\cos x + 2\sin x$ and the corresponding value of x in the range $0 \leq x \leq 360$.

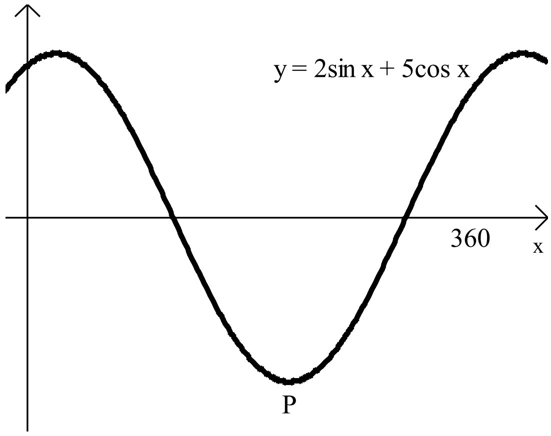
21. (a) Express $\sqrt{3}\cos x - \sin x$ in the form $k\cos(x + a)$ where $k > 0$ and $0 \leq x \leq 360$

(b) Hence solve the equation $\sqrt{3}\cos x - \sin x = -1$ for $0 \leq x \leq 360$.

22. (a) Express $\cos x + \sqrt{3}\sin x$ in the form $k\cos(x - a)$ where $k > 0$ and $0 \leq a \leq 360$.

(b) Hence sketch the graph of $y = \cos x + \sqrt{3}\sin x$ for $0 \leq x \leq 360$.

23. Part of the graph of $y = 2\sin x + 5\cos x$ is shown in the diagram.



(a) Express $2\sin x + 5\cos x$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \leq a \leq 360$.

(b) Find the coordinates of the minimum turning point P.

24. Simplify

(a) $\log_9 12 + \log_9 6 - \log_9 8$ (b) $\frac{2}{3}\log_{10} 8 - \frac{1}{4}\log_{10} 16 + \log_{10} 50$

25. Solve for $x > 0$

(a) $\log_2 x + \log_2 (x - 6) = 4$ (b) $\log_3 4x - \log_3 (x + 1) = 1$

26. Find x in each of the following ($x > 0$)

(a) $2\log_x 4 + \log_x 2 = 5$ (b) $\frac{3}{4}\log_x 81 - 2\log_x 8 = 3$

27. A curve has equation $y = \log_2 (x + 4) - 3$.
Find the coordinates of the points where this curve cuts the x and y axes.

28. The mass, M grams, of a radioactive isotope after a time of t years, is given by the formula $M = M_o e^{-kt}$ where M_o is the initial mass of the isotope.

In 8 years a mass of 40 grams of the isotope is reduced to 36 grams.

(a) Calculate k.

(b) Calculate the half-life of the substance i.e. the time taken for half the substance to decay.

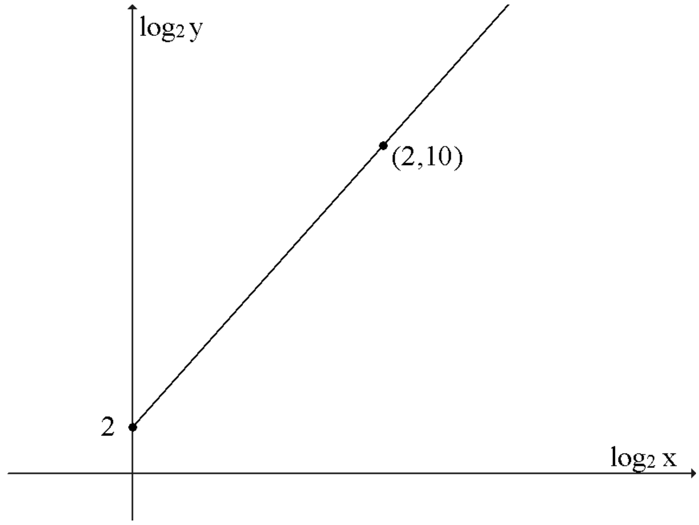
29. The value, V (£million), of an aeroplane is given by the formula $V = 3.5e^{-0.095t}$ where t is the number of years after the aeroplane is put into service..

(a) Calculate the value of the aeroplane when it was built.

(b) How long, to the nearest year, will it take for the aeroplane to fall to 40% of its original value?

30. The graph opposite illustrates the law $y = kx^n$.

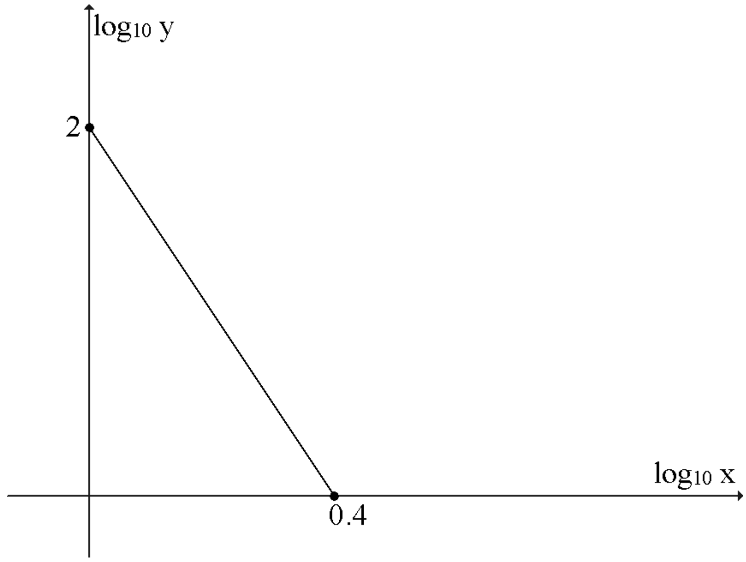
Find the values of k and n.



31. The graph opposite illustrates the law $y = ax^b$.

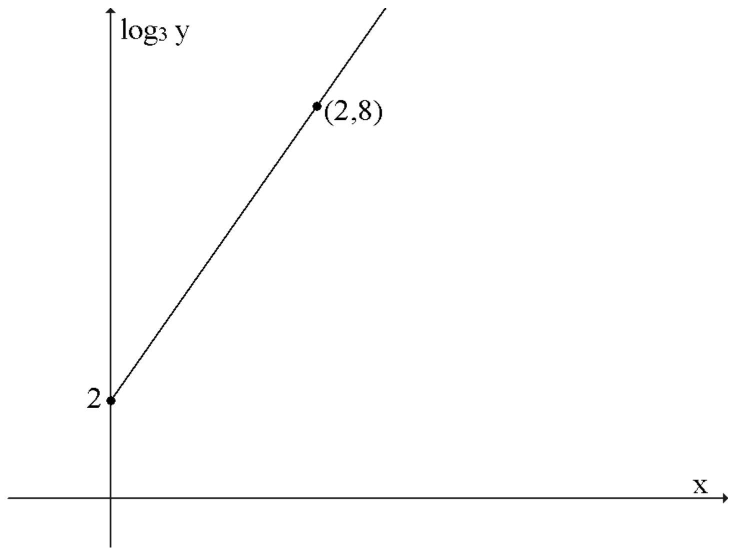
The line passes through the points (0,2) and (0.4,0).

Find the values of a and b.



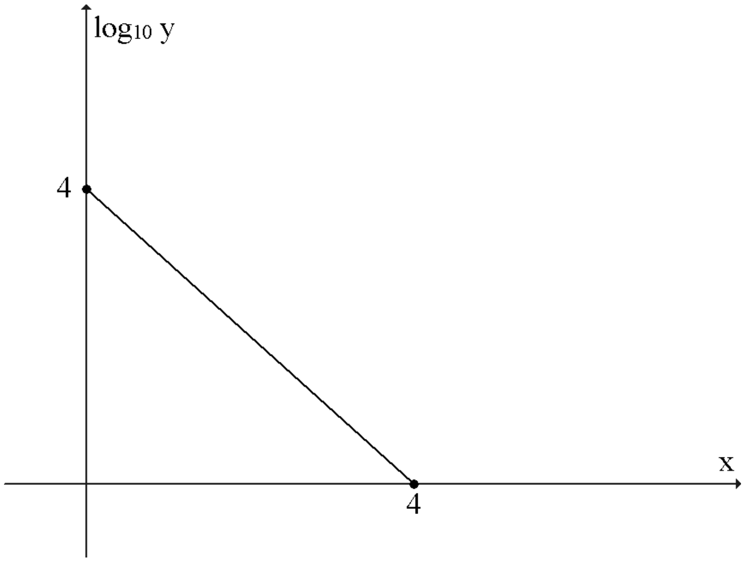
32. The graph opposite illustrates the law $y = ab^x$.

Find the values of a and b.



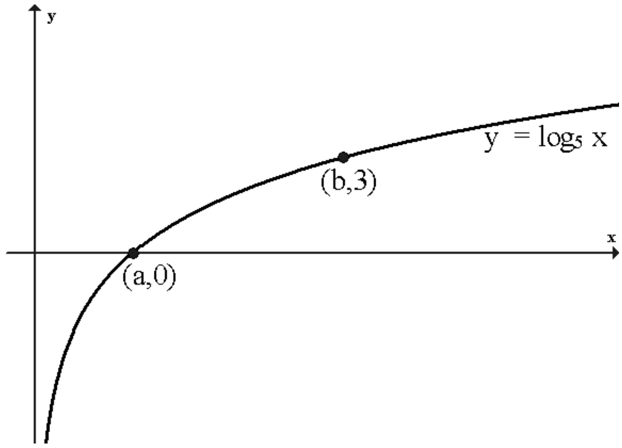
33. The graph opposite illustrates the law $y = kb^x$.

Find the values of k and b.



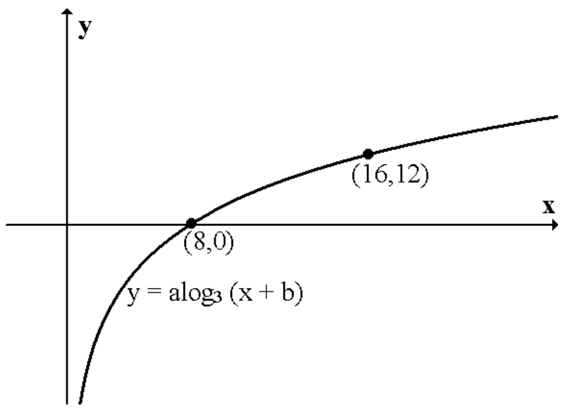
34. The diagram shows part of the graph of $y = \log_5 x$.

(a) Find a and b.
(b) Sketch the graph of $y = \log_5 5x$.
(c) Sketch the graph of $y = \log_5 x^2$
(d) Sketch the graph of $y = \log_5 \frac{1}{x}$.



35. The diagram opposite shows the graph of $y = a\log_3 (x + b)$.

Find the values of a and b.



36. The diagram shows the graph of $y = \log_b (x - a)$.

Find a and b.