Logs/Exp Past Papers Unit 3 Outcome 3

Multiple Choice Questions

Each correct answer in this section is worth two marks.

1. Solve $\log_b x - \log_b 7 = \log_b 3$ for $x > 0$.

   A. $x = 21$

   B. $x = 10$

   C. $x = \frac{7}{3}$

   D. $x = \frac{3}{7}$

[END OF MULTIPLE CHOICE QUESTIONS]

Written Questions
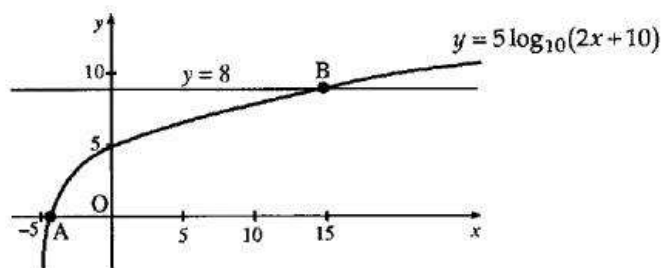
[SQA]   2. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

[SQA]   3. Given $x = \log_5 3 + \log_5 4$, find algebraically the value of x. **4**

[SQA]   4. Find x if $4\log_x 6 - 2\log_x 4 = 1$. **3**

[SQA]   5. Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3(x-2) + 1$ intersects the x-axis. **3**
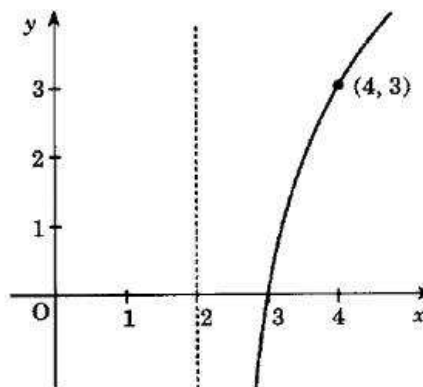
[SQA]   6. Part of the graph of $y = 5\log_{10}(2x+10)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B.

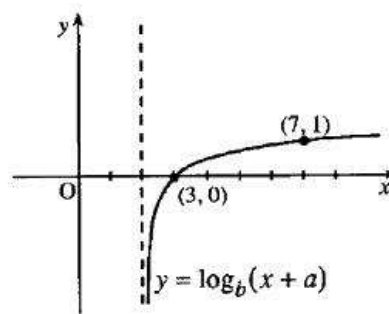Find algebraically the x-coordinates of A and B. **4**

[SQA] 7. **The diagram shows a sketch of the graph of**
$y = f(x)$ **where** $f(x) = a\log_2(x-b)$.
Find the values of a **and** b.

3

[SQA] 8. **The diagram shows part of the graph of** $y = \log_b(x+a)$.
Determine the values of a **and** b.

3

(7, 1)

(3, 0)

$y = \log_b(x+a)$

[SQA] 9. **The diagram shows a sketch of part of the graph of** $y = \log_5 x$.

(a) Make a copy of the graph of $y = \log_5 x$.
On your copy, sketch the graph of $y = \log_5 x + 1$.
Find the coordinates of the point where it crosses the x-axis.

3

(5,1)

(1, 0)

(b) Make a second copy of the graph of $y = \log_5 x$.

On your copy, sketch the graph of $y = \log_5 \dfrac{1}{x}$.
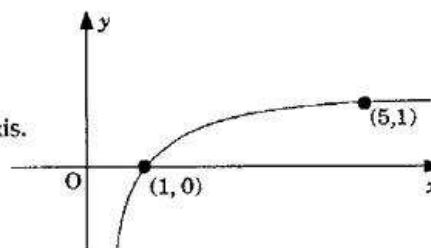
2

[SQA] 10. Medical researchers studying the growth of a strain of bacteria observe that the number of
bacteria, present after t hours, is given by the formula $N(t) = 40e^{1.5t}$.

(a) State the number of bacteria present at the start of the experiment. 1
(b) How many minutes will the bacteria take to double in number? 4

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Higher Mathematics

[SQA]   **11.** A medical technician obtains this print-out of a wave form generated by an oscilloscope. The technician knows that the equation of the first branch of the graph (for $0 \leq x \leq 3$) should be of the form $y = ae^{kx}$.
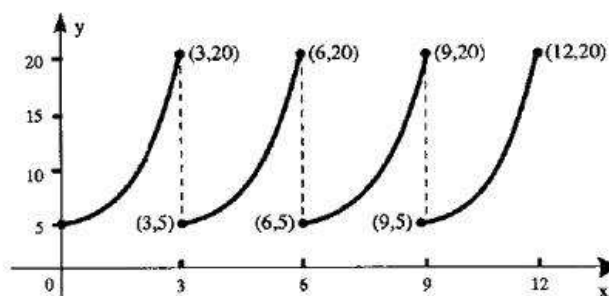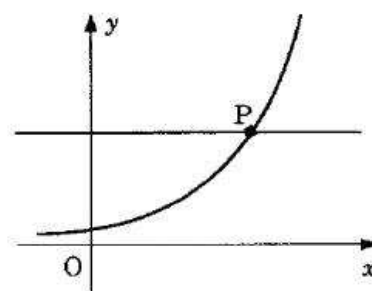

(a) Find the values of a and k.   **4**

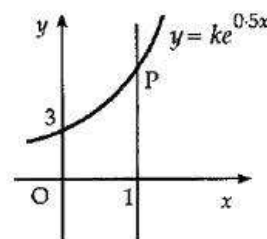(b) Find the equation of the second branch of the curve (i.e. for $3 \leq x \leq 6$).   **1**

[SQA]   **12.** The diagram shows part of the graph with equation $y = 3^x$ and the straight line with equation $y = 42$. These graphs intersect at P.

Solve algebraically the equation $3^x = 42$, and hence write down, correct to 3 decimal places, the coordinates of P.   **4**

[SQA]   **13.** The diagram shows part of the graph of $y = ke^{0.5x}$.

(a) Find the value of k.   **1**

(b) The line with equation $x = 1$ intersects the graph at P. Find the coordinates of the point P.   **2**

[SQA]   **14.** Two sound intensities P_1 and P_2 are said to differ by n decibels when $n = 10\log_{10}\left(\dfrac{P_2}{P_1}\right)$ where P_1 and P_2 are measured in phons and $P_2 > P_1$.

Rustling leaves have a typical sound intensity of 30 phons. If the sound intensity of a fire alarm siren is 6·5 decibels gretaer than rustling leaves, what is the sound intensity of the fire alrm system, measured in phons?   **3**

[SQA]   **15.** Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0 e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.

If it takes one and a half hours for the area of the forest fire to double, find the value of the constant k.   **3**

[SQA] 16. A mug of tea cools according to the law $T_t = T_0 e^{-kt}$ where T_0 is the initial temperature and T_c is the temperature after t minutes. All temperatures are in °C.

(a) A particular mug of tea cooled from boiling point (100°C) to 75°C in a quarter of an hour. Calculate the value of k.   **3**

(b) By how many degrees will the temperature of this tea fall in the next quarter of an hour?   **2**

[SQA] 17. (a) A tractor tyre is inflated to a pressure of 50 units. Twenty-four hours later the pressure has dropped to 10 units.

If the pressure, P_t units, after t hours is given by the formula $P_t = P_0 e^{-kt}$, find the value of k, to three decimal places.   **(5)**

(b) The tyre manufacturer advises that serious damage to the tyre will result if it is used when the pressure drops below 30 units.

If the farmer inflates the tyre to 50 units and drives the tractor for four hours, can the tractor be driven further without inflating the tyre and without risking serious damage to the tyre?   **(4)**

[SQA] 18. The amount A grams of a radioactive substance at time t minutes is given by $A = A_0 e^{-kt}$ where A_0 is the initial amount of the substance and k is a constant.
In 3 minutes, 10 grams of the substance Bismuth are reduced to 9 grams through radioactive decay.

(a) Find the value of k.   **3**

The half-life of a substance is the length of time in which half the substance decays.

(b) Find the half-life of Bismuth.   **2**

[SQA] 19. (a) For a particular radioactive substance the mass m (in grams) at time t (in years) is given by
$$m = m_0 e^{-0.02t}$$
where m_0 is the original mass.

If the original mass is 500 grams, find the mass after 10 years.   **(2)**

(b) The half-life of any material is the time taken for half of the mass to decay.
Find the half-life of this substance.   **(3)**

(c) Illustrate **ALL** of the above information on a graph.   **(3)**

[SQA] 20. The radioactive element carbon-14 is sometimes used to estimate the age of organic remains such as bones, charcoal, and seeds.

Carbon-14 decays according to a law of the form $y = y_0 e^{kt}$ where y is the amount of radioactive nuclei present at time t years and y_0 is the initial amount of radioactive nuclei.

(a) The half-life of carbon-14, i.e. the time taken for half the radioactive nuclei to decay, is 5700 years. Find the value of the constant k, correct to 3 significant figures. **(3)**

(b) What percentage of the carbon-14 in a sample of charcoal will remain after 1000 years? **(3)**

[SQA] 21. The intensity I_t of light is reduced as it passes through a filter according to the law $I_t = I_0 e^{-kt}$ where I_0 is the initial intensity and I_t is the intensity after passing through a filter of thickness t cm. k is a constant.

(a) A filter of thickness 4 cm reduces the intensity from 120 candle-power to 90 candle-power. Find the value of k. **4**

(b) The light is passed through a filter of thickness 10 cm. Find the percentage reduction in its intensity. **3**

[SQA] 22. The results of an experiment give rise to the graph shown.

(a) Write down the equation of the line in terms of P and Q. **2**
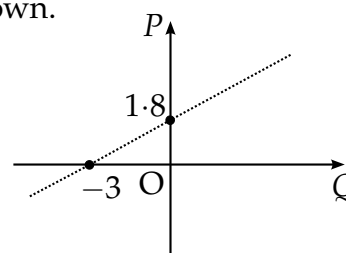
It is given that $P = \log_e p$ and $Q = \log_e q$.

(b) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

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[SQA] 23. When the switch in this circuit was closed, the computer printed out a graph of the current flowing (I microamps) against the time (t seconds).This graph is shown in fig. 1.
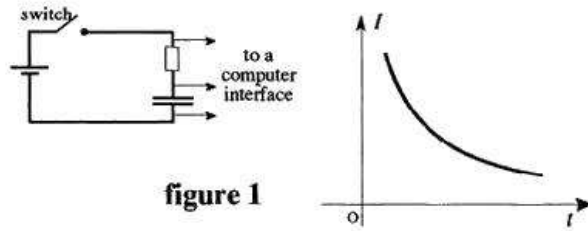
figure 1

In order to determine the equation of the graph shown in figure 1, values of $\log_e I$ were plotted against $\log_e t$ and the best fitting straight line was drawn as shown in figure 2.
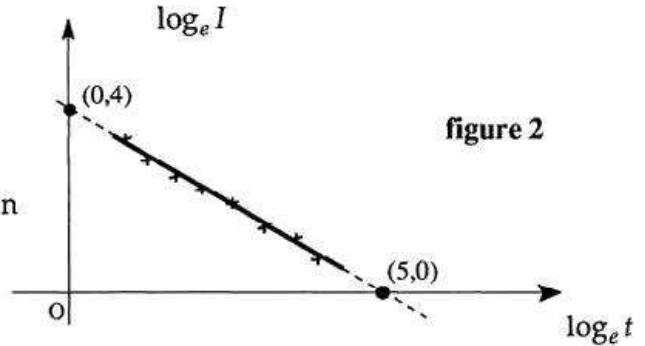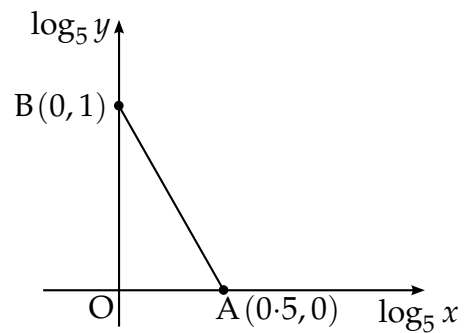
$\log_e I$

(0,4)

figure 2

(5,0)

$\log_e t$

(a) Find the equation of the line shown in figure 2 in terms of $\log_e I$ and $\log_e t$. (3)

(b) Hence or otherwise show that I and t satisfy a relationship of the form $I = kt^r$ stating the values of k and r. (4)

[SQA] 24. The graph illustrates the law $y = kx^n$
If the straight line passes through $A(0{\cdot}5, 0)$ and $B(0, 1)$, find the values of k and n.

$\log_5 y$

$B(0, 1)$

O $A(0{\cdot}5, 0)$ $\log_5 x$

4

[SQA] 25. As shown in the diagram, a set of experimental results gives a straight line graph when $\log_{10} y$ is plotted against $\log_{10} y$. The straight line passes through (0, 1) and has a gradient of 2.

Express y in terms of x.

$\log_{10} y$

(0,1)

O $\log_{10} x$

6

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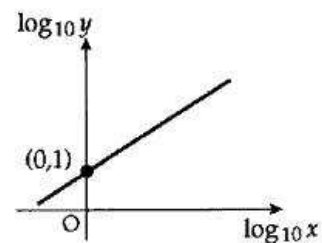
[SQA] 26. (*a*) The variables x and y arc conncctcd by a relationship of the form $y = ae^{bx}$ where a and b are constants. Show that there is a linear relationship between $\log_e y$ and x. **(3)**

(*b*) From an experiment some data was obtained. The table shows the data which lies on the line of best fit.

| x	3.1	3.5	4.1	5.2
y	21 876	72 631	439 392	11 913 076

The variables x and y in the above table are connected by a relationship of the form $y = ae^{bx}$. Determine the values of a and b. **(6)**

[SQA] 27. The point P (p, k) lies on the curve with equation $y = \ln x$.

The point Q (q, k) lies on the curve with equation $y = \frac{1}{2}\ln x$.

Find a relationship between p and q and hence find q when $p = 5$. **4**

[SQA] 28. Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

X ($=\log_e x$)	Y ($=\log_e y$)
2.10	7.00
2.31	7.60
2.40	7.92
2.65	8.70
2.90	9.38
3.10	10.00
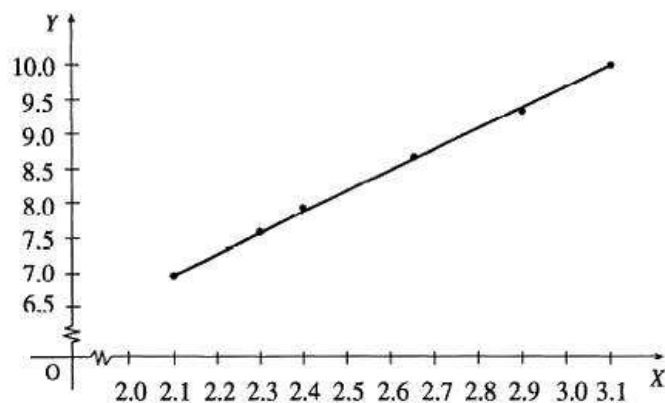
A graph was drawn and is shown above.

O (a) Find the equation of the line in the form $Y = mX + c$. **(3)**

x (b) Hence find the values of the constants a and b in the relationship $y = ax^b$. **(4)**
y

[END OF WRITTEN QUESTIONS]