Support Materials
National Assessment Bank pack

Mathematics Higher

Mathematics 1 D321 12/NAB001

SCQF Level 6

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Contents

Section 1 – Performance Criteria

Performance Criteria summary

Outcome 1

Use the properties of the straight line.

Performance Criteria

(a) Determine the equation of a straight line given two points on the line or one point and the gradient.
(b) Find the gradient of a straight line using $m = \tan\theta$.
(c) Find the equation of a line parallel to and a line perpendicular to a given line.

Outcome 2

Associate functions and graphs.

Performance Criteria

(a) Sketch and identify related graphs and functions.
(b) Identify exponential and logarithmic graphs.
(c) Find composite functions of the form $f(g(x))$ given $f(x)$ and $g(x)$.

Outcome 3

Use basic differentiation.

Performance Criteria

(a) Differentiate a function reducible to a sum of powers of x.
(b) Determine the gradient of a tangent to a curve by differentiation.
(c) Determine the coordinates of the stationary points on a curve and justify their nature using differentiation.

Outcome 4

Define and interpret mathematical models of situations involving recurrence relations.

Performance Criteria

(a) Define and interpret a recurrence relation in the form $u_{n+1} = au_n + b$ (a and b constants) in a mathematical model.
(b) Find and interpret the limit of the sequence generated by a recurrence relation in a mathematical model (where the limit exists).

Section 2 – Instrument of Assessment

Unit Assessment – Mathematics 1 (Higher)

Outcome 1 **Marks**

1 A line passes through the points (4, –3) and (–1,2)
Find the equation of this line. **2**

2 A line makes an angle of 65^o with the positive direction of the x-axis, as shown in Diagram 1.

The scales on the axes are equal.

Find the gradient of the line giving your answer correct to 3 significant figures.

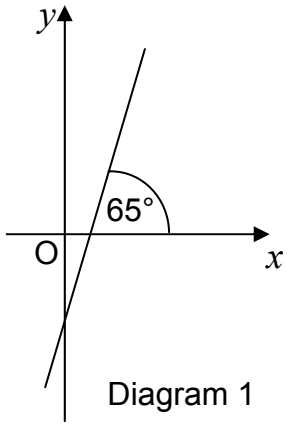


Diagram 1

2

3 A line L has equation $y = 2x + 1$
Write down the gradient of a line which is:

(a) parallel to L **1**

(b) perpendicular to L. **1**

Outcome 2 **Marks**

4 The graph of a cubic $y = f(x)$ is shown in Diagram 2.

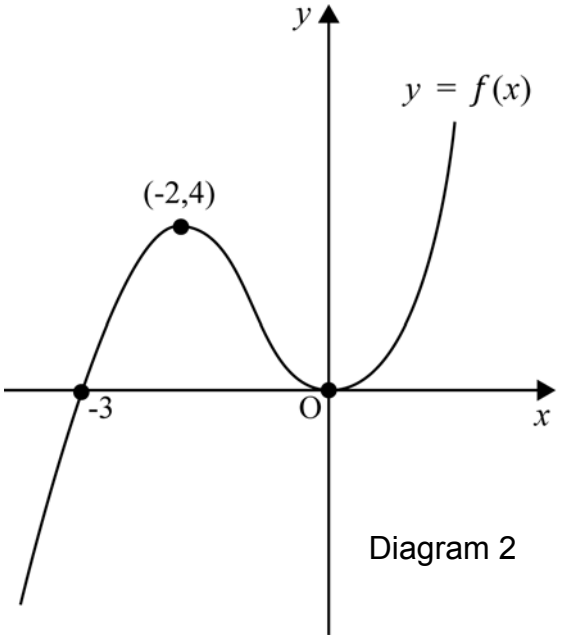


Diagram 2

On separate diagrams sketch the graphs of:

(a) $y = -f(x)$ **2**

(b) $y = f(x + 2)$ **2**

5 The graphs with equations $y = 2^x$ and $y = a^x$ are shown in Diagram 3.

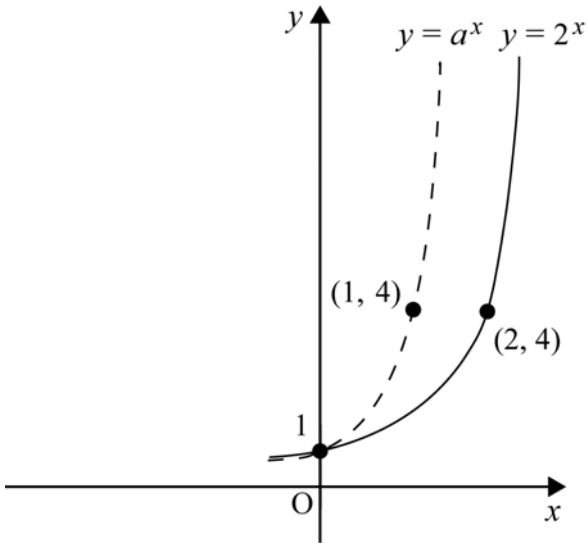


Diagram 3

If the graph with equation $y = a^x$ passes through the point (1,4), find the value of a **1**

Marks

6 The graphs of $y = 10^x$ and its inverse function are shown in Diagram 4.

Write down the equation of the inverse function. **1**

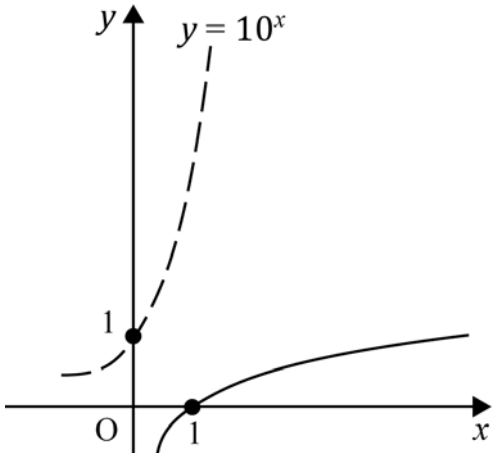


Diagram 4

7 Functions f and g are defined on suitable domains by $f(x) = x^2$ and $g(x) = 2x + 1$.
Obtain an expression for $f(g(x))$. **2**

Outcome 3

8 Given $y = \frac{5}{x^3}\ x \neq 0$, find $\frac{dy}{dx}$ **2**

9 A sketch of the curve with equation $y = x^2 - 8x + 14$ is shown in Diagram 5.

A tangent has been drawn at the point P(6,2)

Find the gradient of the tangent at P.

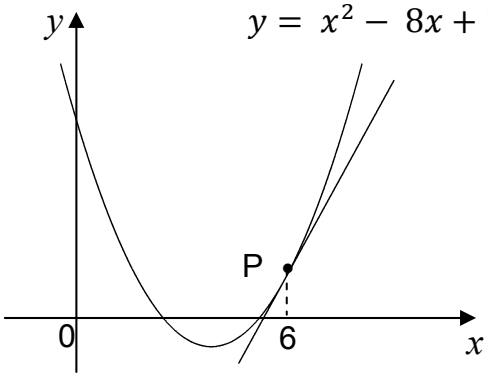


Diagram 5 **3**

10 A curve has equation $y = \frac{1}{3}x^3 - 2x^2 + 3x + 1$

Using differentiation, find the coordinates of the stationary points on this curve and determine their nature. **6**

Outcome 4 **Marks**

11 In a pond, one quarter of the existing tadpoles are eaten by predators each day but during the night 1000 tadpoles are hatched.

There are u_n tadpoles at the start of a particular day.

(a) Write down a recurrence relation for u_{n+1} the number of tadpoles at the start of the next day. **1**

(b) It is known that the pond cannot sustain more than 2500 tadpoles at any one time.

(i) Find the limit of the sequence generated by this recurrence relation as $n \to \infty$.

(ii) In the long term, can the pond sustain the number of tadpoles? **3**

End of assessment

Section 3 – Marking information

Test specification grid

The grid below shows how the Outcomes and Performance Criteria are assessed in this Unit assessment.

Topic	PC	Question	Marks	Total	Threshold
Straight line	1(a) 1(b) 1(c)	1 2 3	2 2 2	**6**	**4**
Functions and graphs	2(a) 2(b) 2(c)	4 5 6 7	4 1 1 2	**8**	**6**
Basic differentiation	3(a) 3(b) 3(c)	8 9 10	2 3 6	**11**	**8**
Recurrence relations	4(a) 4(b)	11 11	1 3	**4**	**3**

Marking information – Mathematics 1 (Higher)

Recommended general marking information

General marking instructions

1 Marks should be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2 Award one mark for each • (bullet point). Each error should be underlined at the point in the working where it first occurs, and not any subsequent stage of the working.

3 The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided the level of difficulty is approximately similar. Where, subsequent to an error, the working is eased, a deduction of marks(s) should be made.

4 As indicated on the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking information.

5 Do not penalise:

- working subsequent to a correct answer
- omission of units (except where marks are awarded for this in the detailed marking instructions)
- legitimate variations in numerical answers
- correct working in the wrong part of a question
- bad form

6 No piece of work should be scored through without careful checking − even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking information – answers which are widely off beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for marks.

7 No marks should be deducted for careless or badly arranged work.

8 Transcription errors – In general, as a consequence of a transcription error, candidates lose the opportunity of gaining the first accuracy or processing mark.

9 Casual errors – In general, as a consequence of a casual error, candidates lose the opportunity of gaining the first accuracy or processing mark.

10 Acceptable alternative methods of solution can only be given the marks specified in the marking information if the question does not stipulate the method candidates are to use to find the solution. In such circumstances, no marks may be awarded even though the candidate may have obtained the correct answer.

11 In general do not penalise the same error twice in the one question.

12 If an answer is scored out and not replaced, the scored out working should be marked where it is legible.

13 If a candidate presents more than one complete solution to a question and it is not clear which is intended as their final attempt, then each attempt should be marked and the lowest mark awarded. It is anticipated that this will be a rare occurrence.

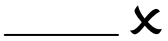
Marking signs and abbreviations

It is recommended that markers use the following signs and abbreviations for marking purposes:

Tick when a piece of working is correct and gains a mark.

A cross-tick should be used to indicate 'correct working' where a mark is awarded as a result of follow through from an error.

A double cross-tick should be used to indicate correct working which is inadequate to score any marks eg incorrect method which is mathematically correct or eased working.



Underline and cross each error especially those where a mark has been lost.

A tilde should be used to indicate a minor transgression which is not being penalised, eg bad form.

Use a roof to show that something is missing such as a crucial step in the working or part of a solution.

B An upper case B should be used to indicate that you have given the candidate the benefit of the doubt and awarded a mark.

E An upper case E should be used to indicate that the candidate has eased the working as a consequence of an error and that marks have been deducted as a result.

Note – In Course assessments, the letters **B** and **E** would not be used.

Outcome 1

Qs	Give 1 mark for each •	Illustrations for awarding each mark
1	$x + y - 1 = 0$	
	•[1] Find gradient •[2] State equation of line	•[1] $m = -1$ •[2] $y-(-3) = -(x-4)$ **or** $y-2 = -(x-(-1))$
Notes: 1 •[2] is still available as follow through from an incorrect gradient. 2 No marks should be deducted as the result of an error subsequent to •[2] being awarded.		
2	$m = 2{\cdot}14$	
	•[1] Use $m = \tan\theta$ with correct angle •[2] State gradient	•[1] tan 65° •[2] $2{\cdot}14$
Notes: 1 •[1] is not available for simply stating the formula $m = \tan\theta$, the correct angle must be substituted for θ. 2 Any answer which rounds correctly to one decimal place, or more accurate, is acceptable for •[2].		
3(a)	2	
	•[1] State parallel gradient	•[1] 2
3(b)	$-\frac{1}{2}$	
	•[2] State perpendicular gradient	•[2] $-\frac{1}{2}$
Notes: 1 There are no follow through marks available in this question, (a) and (b) are independent of each other.		

Outcome 2

Qs	Give 1 mark for each •	Illustrations for awarding each mark
4(a)	Graph reflected in x −axis.	
	•[1] Determine required transformation	•[1] Reflection in x −axis.
	•[2] State coordinates of points on graph	•[2] Roots at $x = 0$ and $x = -3$ and $(-2, -4)$ clearly identified on graph.
4(b)	Graph translated two units to left.	
	•[3] Determine required transformation	•[3] Translation parallel to x-axis.
	•[4] State coordinates of points on graph	•[4] Roots at $x = -2$ and $x = -5$ and $(-4,4)$ clearly identified on graph.
Notes: 1	•[2] is only available as follow through as a consequence of a reflection.	
2	•[4] is only available as follow through from a translation parallel to the x-axis.	
5	$a = 4$	
	•[1] Interpret equation from graph	•[1] $a = 4$
Notes: 1	Accept $y = 4^x$ for •[1] to be awarded.	
2	4 on its own without any other working or evidence does not earn any marks.	
6	$y = \log_{10} x$	
	•[1] State equation of inverse in logarithmic form	•[1] $y = \log_{10} x$
Notes: 1	The answer must be given in the form of a logarithmic equation. Do not accept $\log_{10} x$.	
2	The base must be clearly stated in the final answer. Do not accept $y = \log x$.	
7	$f(g(x)) = (2x + 1)^2$	
	•[1] Interpret composition	•[1] $f(2x + 1)$
	•[2] Complete interpretation of composition	•[2] $(2x + 1)^2$
Notes: 1	For those who find $g(f(x))$ leading to $2x^2 + 1$ then •[2] should be awarded.	
2	There are no marks available for any other interpretation of $f(g(x))$.	

Outcome 3

Qs	Give 1 mark for each •	Illustrations for awarding each mark
8	$\frac{dy}{dx} = -\frac{15}{x^4}$	
	•[1] Express in differentiable form •[2] Differentiate a negative power	•[1] $5x^{-3}$ •[2] $-15x^{-4}$
Notes: 1 •[2] is only available for differentiating a negative power. 2 The correct answer only without working should receive full credit.		
9	$m_{tangent} = 4$	
	•[1] Know to differentiate •[2] Differentiate •[3] Evaluate gradient	•[1] $\frac{dy}{dx} = ..$ stated or implied by •[2] •[2] $2x - 8$ •[3] $m = 4$
Notes: 1 •[3] is only available if an attempt to find the gradient is made from differentiation.		

Qs	Give 1 mark for each •	Illustrations for awarding each mark
10	Max T.P. at $(1,\frac{7}{3})$ and min T.P. at (3,1)	
	$\bullet^1$ Differentiate $\bullet^2$ Set derivative equal to zero $\bullet^3$ Solve $\frac{dy}{dx}=0$ $\bullet^4$ Find corresponding y values $\bullet^5$ Justification $\bullet^6$ State conclusions	$\bullet^1$ x^2-4x+3 $\bullet^2$ $\frac{dy}{dx}=0$ $\bullet^3$ $x=1$ and 3 ($\bullet^3$ $\bullet^4$) $\bullet^4$ $y=\frac{7}{3}$ and 1 $\bullet^5$ / $\bullet^6$ table ($\bullet^5$ $\bullet^6$): x | …..1….. | .....3..... $\frac{dy}{dx}$ | + 0 – | – 0 + Max at $x=1$ | Min at $x=3$

Notes: 1 The "$=0$" at $\bullet^2$ stage must appear at least once before the $\bullet^3$ stage.

2 $\bullet^3$ is only available as a consequence of solving $\frac{dy}{dx}=0$.

3 The nature table must reflect previous working from $\bullet^3$.

4 $\bullet^3$ is only available for two solutions. If extra solution appears $\bullet^3$ is not available.

5 As shown ($\bullet^3$ and $\bullet^4$) and ($\bullet^5$ and $\bullet^6$) can be marked horizontally or vertically.

6 $\bullet^4$ is only available as follow through if a quadratic equation has been solved.

7 The use of the second derivative is an acceptable strategy for $\bullet^5$.

Outcome 4

Qs	Give 1 mark for each •	Illustrations for awarding each mark
11(a)	$u_{n+1} = \frac{3}{4}u_n + 1000$	
	•[1] State recurrence relation	•[1] $u_{n+1} = \frac{3}{4}u_n + 1000$ or $u_{n+1} = 0 \cdot 75u_n + 1000$
11(b)	Limit 4000 and no pond cannot sustain tadpole numbers	
	•[2] Know how to find limit	•[2] $l = \frac{3}{4}l + 1000$ or $l = 0 \cdot 25l + 1000$
	•[3] Process limit	•[3] $l = 4000$
	•[4] Compare and state conclusion	•[4] $4000 > 2500$ so pond cannot sustain tadpole numbers.

Notes: **In (a)** . . .

1 The only answers that should be awarded •[1] are as given above.

In (b) . . .

2 For •[2] accept $l = \frac{b}{1-a}$ with numerical substitutions for a and b.

3 Any calculations based on any other formula masquerading as a limit rule cannot gain •[2] and •[3], however •[4] is still available as follow through where an appropriate comparison and comment are made.

4 •[3] and •[4] are only available for a lying in the interval $-1 < a < 1$.

5 •[4] is only available as a consequence of comparing the limit evaluated with 4000 and making an appropriate comment.