RECURRENCE RELATIONS   SET 2

1   The Scottish Bank pays 9% compound interest p.a. on it's Super Saver account. A customer
     opens such an account with a deposit of £500. If B_n is the balance after n years:
     (a)   Write down a recurrence relation.
     (b)   Calculate B_1, B_2, B_3, B_4.
     (c)   Write down a formula for B_n.
     (d)   If no further deposits are made, after how many years will the value of the interest
            gained have exceeded the value of the initial deposit?

2   State whether the following recurrence relations have a limit and if so determine algebraically
     this limit.
     (a) $u_0 = 8,$ $u_{n+1} = 0.25u_n + 4$ (b) $u_0 = 2,$ $u_{n+1} = 12 - 1.5u_n$

3   A recurrence relation is defined by $u_{n+1} = pu_n + q$ where $-1 < p < 1$ and $u_0 = 12$
     (a) If $u_1 = 15$ and $u_2 = 16$ find the values of p and q
     (b) Find the limit of this recurrence relation as $n \to \infty$

4   A hospital patient is put on medication which is taken once per day. The dose is 35mg and
     each day the patient's metabolism burns off 70% of the drug in her system. It is known that if
     the level of the drug in the patients system reaches 54mg then the consequences could be
     fatal. Is it safe for the patient to take the medication indefinitely?

5   The brake fluid reservoir in a car is leaky. Each day it loses 3% of its contents. To
     compensate for this daily loss the driver "tops up" once per week with 50ml of fluid. For
     safety reasons the level of fluid in the reservoir should always be between 200ml & 260ml.
     Initially the fluid level is 255ml.
     (a)   Find a recurrence relation to describe the above.
     (b)   Determine the fluid levels after 1 week and 4 weeks.
     (c)   Is the process effective in the long run?