Functions 2

1. $f(x) = 2x - 5$ and $g(x) = x^2 + 1$. Find a formula for $f(g(x))$.

2. $f(x) = x^2 - 2$. Find a formula for $f(f(x))$.

3. The functions f and g are defined on suitable domains with

$$f(x) = \frac{2}{x+1} \quad \text{and } g(x) = \frac{2x+4}{x}$$

   Show that $g(f(x)) = 2x + 4$.

4. $f(x) = \frac{4}{3x+1} \quad x \neq -\frac{1}{3} \quad$ and $\quad g(x) = 4x - 3$

   (a) Find a formula for $f(g(x))$.
   (b) State a suitable domain for $f((g(x))$.

5. The functions f and g are defined on suitable domains with

$$f(x) = \frac{1}{x^2 - 1} \quad \text{and} \quad g(x) = x + 1$$

   (a) $h(x) = g(f(x))$. Find an expression for $h(x)$. Give your answer as a single fraction.
   (b) State a suitable domain for $h(x)$.

6. $f(x) = \frac{1}{2x+4} \quad x \neq -2 \quad$ and $g(x) = \frac{1}{x-1} \quad x \neq 1$

   (a) Find a formula for $f(g(x))$.
   (b) State a suitable domain for $f(g(x))$.

7. The function f, defined on a suitable domain, is $f(x) = \frac{x}{x-1}$

   (a) Find a formula for $f(f(x))$.
   (b) What can you say about the function f?

8. $f(x) = 2\sin x$ and $g(x) = (x + \frac{\pi}{2})$

    (a) Given $k(x) = f(g(x))$, find a formula for $k(x)$.
    (b) Solve the equation $k(x) = -1$, for $0 \le x \le 2\pi$

9. $f(x) = 2x - 1$ and $g(x) = x^2 + x$.

    (a) Find a formula for $g(f(x))$ in its simplest form.
    (b) $h(x) = g(f(x)) - f(g(x))$. Find a formula for $h(x)$.
    (c) Solve the equation $h(x) = 7$.

10. $f(x) = x^2 - 2$ and $g(x) = 2x + 1$

    (a) Find expressions for $f(g(x))$ and $g(f(x))$.
    (b) There is only one value of x for which $f(g(x)) = g(f(x))$, find this value of x.

11. $f(x) = 3x - 10$ $g(x) = 4 - 2x$ $h(x) = \frac{1}{6}(2 - x)$

    (a) $k(x) = f(g(x))$. Find $k(x)$.
    (b) Find a formula for $h(k(x))$.
    (c) What is the connection between h and k?

12. $f(x) = 3x - 2$ and $g(x) = 3x + 2$

    (a) Find formulae for $f(g(x))$ and $g(f(x))$.
    (b) Find the least value of the product $f(g(x)) \times g(f(x))$.

13. $f(x) = x^2 + 1, x \ge 0$ and $g(x) = \sqrt{x - 1}, x \ge 1$

    (a) Sketch f(x) for the given domain.
    (b) Find an expression for $f(g(x))$.
    (c) Hence, or otherwise sketch g(x) for the given domain.

14. $f(x) = 2x^2 + 1, x \ge 0$ and $g(x) = \sqrt{\frac{1}{2}(x - 1)}, x \ge 1$.

    (a) Sketch f(x) for the given domain.
    (b) Find an expression for $g(f(x))$.
    (c) Hence, or otherwise, sketch the graph of g(x) for the given domain.