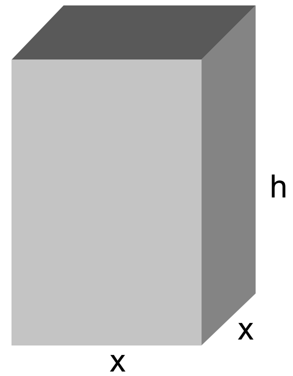
Maxima and Minima

1. A solid cuboid measures x units by x units by h units.
   The volume of this cuboid is 125 units^3.

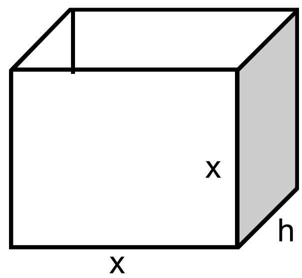


   (a) Show that $h = \dfrac{125}{x^2}$

   (b) Show that the surface area of this cuboid is given by
   $A(x) = 2x^2 + \dfrac{500}{x}$.

   (c) Find the value of x such that the surface area is minimised.

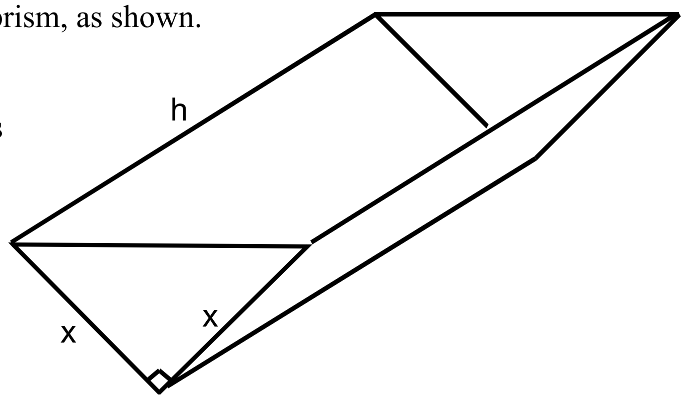
2. An open cuboid (i.e no top) has measurements x units by x units by h units. Its volume is 288 units^3.



   (a) Show that the surface area of this cuboid is $A(x) = 2x^2 + \dfrac{864}{x}$.

   (b) Find the dimensions of this cuboid if the surface area is to be minimised.

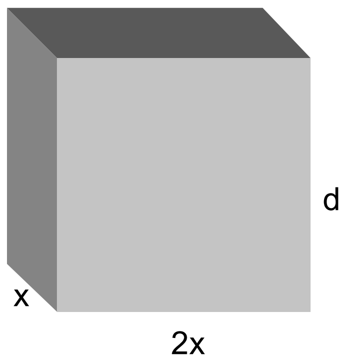
3. An open trough is in the shape of a triangular prism, as shown.
   The trough has a capacity of 256 000 cm^3.



   (a) Show that the surface area of the trough is
   $A(x) = x^2 + \dfrac{1\,024\,000}{x}$

   (b) Find the value of x such that this surface area is as small as possible.

4. The diagram shows a solid cuboid. The surface area of this cuboid is 600 cm^2.

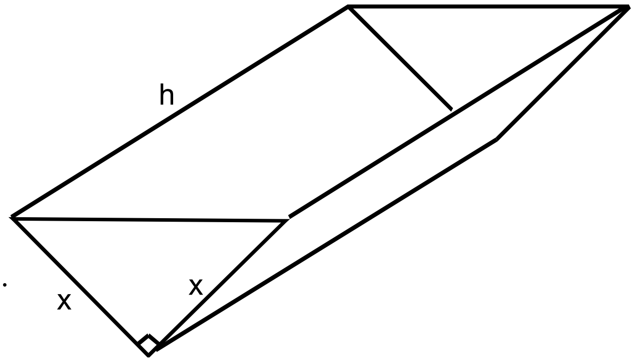


   (a) Show that $d = \dfrac{100}{x} - \dfrac{2x}{3}$

   (b) Show that the volume of the cuboid is given by
   $V = 200x - \dfrac{4}{3}x^3$.

   (c) Find the value of x which maximises this volume.

5. An open trough, as shown, has surface area 4 m^2.



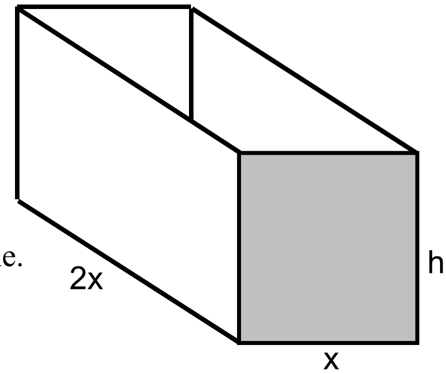
(a) Show that $h = \frac{2}{x} - \frac{x}{2}$.

(b) Show that the volume of the trough is

$$V = x - \frac{x^3}{4}.$$

(c) Show that for a maximum volume $x = \frac{2}{\sqrt{3}}$.

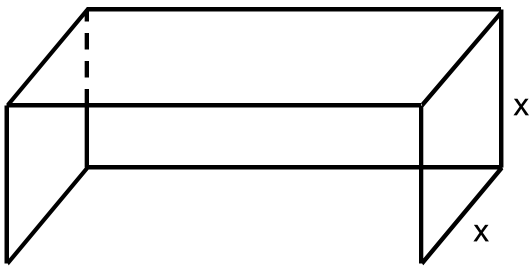
6. An open open cuboid is shown opposite.
The surface area of this cuboid is 12 units^2.



(a) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x(6 - x^2)$

(b) Find the exact value of x which gives a maximum volume.

7. A wind shelter, as shown, has a back, top and two square sides.
The total amount of canvas used to make the shelter is 96 m^2.

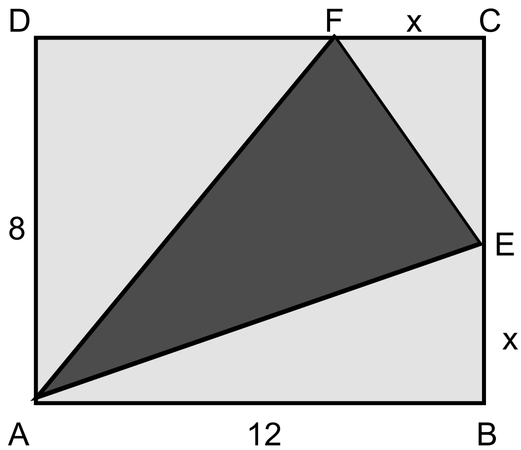


(a) Show that the volume of the shelter is

$$V = x(48 - x^2)$$

(b) Find the dimensions of the shelter of maximum volume, justifying your answer.

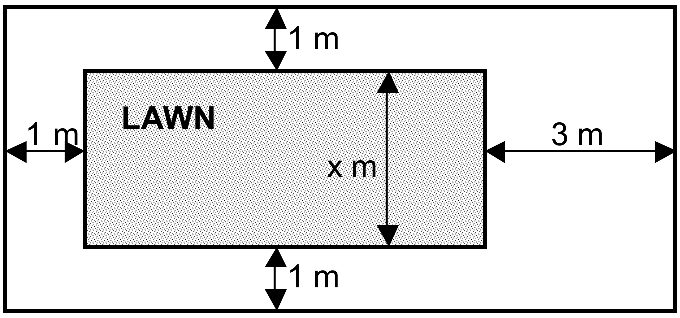
8. A flag consists of red triangle on a yellow rectangular background. In the yellow rectangle ABCD, AB = 12 and AD = 8. EB = CF = x.



(a) Show that the area H, of the red triangle is Given by $H(x) = 48 - 6x + \frac{1}{2}x^2$

(b) Find the biggest possible area of the triangle.

9. A rectangular garden is laid out as shown in the diagram with a rectangular lawn of area 50 square metres surrounded by a border. The lawn has breadth x metres.



(a) If the total area of the garden is A square metres, show that $A = 58 + 4x + \frac{100}{x}$.

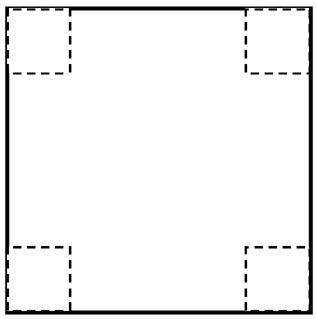
(b) Find the value of x which minimises the area of the garden.

10. When a ship is travelling at a speed of v kilometres per hour it uses fuel at a rate of $(1 + 0.0005v^3)$ tonnes per hour.

(a) Prove that the total amount of fuel used on a voyage of 5000 km at a speed of v kilometres per hour is $A = \frac{5000}{v} + 2.5v^2$ tonnes.

(b) Find the speed which minimises the amount of fuel used and the amount of fuel used at this speed.

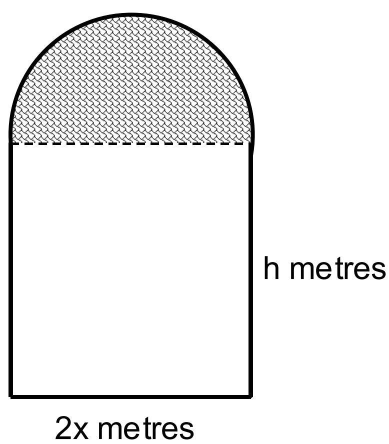
11. A square piece of card of side 50 cm has a square of side x cm cut from each corner. An open box is formed by turning up the sides.



(a) Show that the volume, V, of the open box is given by $V = 2500x - 200x^2 + 4x^3$.

(b) Find the maximum volume of the box, justifying your answer.

. 12. A window is in the shape of a rectangle surmounted by a semi-circle. The glass in the semi-circular part is stained glass which lets in one unit of light per m^2 and the glass in the rectangular part is clear which lets in 2 units of light per m^2.



(a) If the perimeter of the window is 10 metres, show that $h = \frac{10 - 2x - \pi x}{2}$.

(b) Hence show that the amount of light, L, let in by the window is $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$.

(c) Find the value of x such that the design of the window lets in the maximum amount of light.